Data Abundance and Asset Price Informativeness

On-Line Appendix

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This note is the on-line appendix for "Data Abundance and Asset Price Informativeness." It contains the proofs for findings mentioned in the paper but not proven there for the sake of brevity. It is not intended for publication. The on-line appendix is organized as follows.

- Section 1 shows that it is optimal for a shallow information speculator to trade at date 1, when he receives his signal, rather than at date 2.
- Section 2 provides the proof of Lemma 2 in the paper.
- Section 3 completes the proof of Proposition 3 (case 2).
- Section 4 shows that $C_{max}(\theta, \alpha_1)$ decreases with α_1 as claimed in the proof of Proposition 4.

1 It is optimal for shallow information speculators to trade at date 1.

Let α_1 and α_2 be the masses of shallow information speculators who choose to trade at dates 1 and at date 2. Assume that $\alpha_1 \leq 1$ since otherwise the expected profit of shallow

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information speculators who trade at date 1 is negative. Thus, this situation cannot arise in equilibrium. In this case, the unique equilibrium prices and actions for speculators who trade at date 1 or 2 are given in Propositions 1 and 2.

Now, suppose that we offer the possibility to shallow information speculators to choose whether to trade at date 1 or 2. In making his decision, each shallow information speculator takes other speculators' decisions as given, i.e., take prices and masses of speculators who trade at dates 1 and 2 as given since each speculator is atomistic. We show below that the optimal decision of a shallow speculator is then to trade at date 1, as assumed in our model.

To see this, consider one shallow information speculator and suppose that $s = 1$. As he is negligible, this investor takes the mass of shallow speculators who trade at date 1 as given and therefore unaffected by his decision to trade at date 1 or 2. If the shallow information speculator trades at date 1 then his expected profit is $\frac{\theta}{2}(1 - \alpha_1)$. If instead he trades at date 2, there are two possibilities.

First, with probability α_1, the price at date 1 reveals the signal owned by the shallow information speculators (s). Thus, $p(1) = \mu(1)$. In this case, if he places an order, the shallow information speculator expects to trade at a price equal to $E(p(2) \mid s = 1) = E(E(V \mid f_2) \mid s = 1) = \mu(1)$, where the first equality comes from eq.(3) and the second from the law of iterated expectations. thus, in this case, if he places a buy or a sell market order at date 2, the shallow information speculator expects zero profit.

Second, with probability $(1 - \alpha_1)$, the price at date 1 is equal to $E(V) = 1/2$. In this case, the shallow information speculator faces the same trading problem at date 2 than at date 1 and therefore, if he trades, he must behave in the same way, i.e., he buys the asset if $s = 1$ and sells it if $s = -1$. Intuitively, this strategy can at least generate a positive expected profit when $u = 1$ since in this case the shallow information speculator just trades as deep information speculators trade.

Suppose first that $\alpha_2 \leq 1$. Then, using the expression for the equilibrium price at date 2 in Proposition 2, we can compute the expected profit of the shallow speculator and we obtain that it is equal to $(1-\alpha_2(2-\theta)^{-1})\frac{\theta}{2}$.[1] Thus, the expected profit of the shallow information speculator if $s=1$ and he chooses to trade at date 2 is $(1-\alpha_2(2-\theta)^{-1})(1-\alpha_1)\frac{\theta}{2}$, which is less than $(1-\alpha_1)\frac{\theta}{2}$. Thus, in this case, it is optimal for the shallow information speculator to trade at date 1.

Now suppose $\alpha_2 \geq 1$. Then, using the expression for the equilibrium price at date 2, we can compute the expected profit of the shallow speculator and we obtain that it is equal to $\frac{(2-\alpha_2)(1-\theta)}{(2-\theta)}\frac{\theta}{2}$. Thus, the expected profit of the shallow information speculator if $s=1$ and he chooses to trade at date 2 is $\frac{(2-\alpha_2)(1-\alpha_1)(1-\theta)}{(2-\theta)}\frac{\theta}{2}$., which again is less than $(1-\alpha_1)\frac{\theta}{2}$. Thus, in this case as well, it is optimal for the shallow information speculator to trade at date 1.

In sum, when $s=1$, we have shown that it was optimal for a shallow information speculator to trade at date 1 rather than at date 2, for any values of α_1 and α_2 in the range relevant for our analysis. A similar reasoning shows that this is also the case when $s=0$.

2 Proof of Lemma 2.

The equilibrium conditions for an interior equilibrium of the market for shallow information are

$$\bar{\pi}_1(\alpha_1^*) = \frac{C_r}{\alpha_1^*},$$

$$\frac{\partial}{\partial \alpha_1}\left[\bar{\pi}_1(\alpha_1) - \frac{C_r}{\alpha_1}\right]_{\alpha_1=\alpha_1^*} = \frac{\partial \bar{\pi}_1}{\partial \alpha_1}(\alpha_1^*) + \frac{C_r}{\alpha_1^{*2}} < 0$$

The last condition ensures that there is no incentive for new speculators to enter. It can be rewritten as

$$\frac{\partial \bar{\pi}_1}{\partial \alpha_1} + \frac{\bar{\pi}_1(\alpha_1^*)}{\alpha_1^*} = \frac{1}{\alpha_1^*}\frac{\partial}{\partial \alpha_1}\left[\alpha_1 \bar{\pi}_1(\alpha_1)\right]_{\alpha_1=\alpha_1^*} < 0$$

[1] Intuitively, the shallow information speculator obtains the same expected profit as a deep information speculator if $s=1$ and $u=1$ because then he trades in the same way and obtains a zero expected profit if $u=0$.

Hence the problem is equivalent to find the solutions of the equation $\alpha_1\bar{\pi}_1(\alpha_1) = C_r$, for which the derivative of $\alpha_1\bar{\pi}_1(\alpha_1)$ with respect to α_1 is negative. The solution will be less than 1, $0 < \alpha_1^* < 1$, if the following equation has a solution in $[0, 1]$

$$\alpha_1\bar{\pi}_1(\alpha_1) = \frac{\theta}{2}\alpha_1(1-\alpha_1) = C_r \Leftrightarrow \alpha_1(1-\alpha_1) = \frac{2C_r}{\theta}$$

The function of $\alpha_1(1-\alpha_1)$ is increasing on $[0, 1/2]$ and decreasing on $[1/2, 1]$. For $\alpha_1 = 1/2$, $\alpha_1(1-\alpha_1)$ is equal to $1/4$, its maximum. Hence if $2C_r/\theta > 1/4$ there is no interior solution, then $\alpha_1^* = 0$. If $2C_r/\theta < 1/4$, the only admissible solution is on the interval $[1/2, 1]$ and is equal to

$$\alpha_1^* = \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{2C_r}{\theta}}$$

3 Complement to the proof of Proposition 3.

Let $\bar{\alpha}_1(\theta) = \frac{(1-\theta)^2+\theta^2}{(1-2\theta)[2(1-\theta)(2-\theta)-1]}$ and $\bar{C}_p(\theta) = \frac{(\theta(1-\theta)^2)(1-2\theta)}{(2-\theta)\left(2(1-\theta)-\frac{1}{2-\theta}\right)^2}$. We first prove the following result.

Lemma 3.1. *For $\theta < 1/2$ and $C_{min}(\theta, \alpha_1) \leq C_p < C_{max}(\theta, \alpha_1)$, $\frac{\partial \alpha_2^*}{\partial \alpha_1} > 0$ if and only if $\theta < \frac{\sqrt{2}-1}{\sqrt{2}}$ and $\alpha_1 > \bar{\alpha}_1(\theta)$ and $C_p > \bar{C}_p(\theta)$.*

Proof. When $\theta < 1/2$, we know from Corollary 1 that $\frac{\partial \alpha_2^*}{\partial \alpha_1} > 0$ if and only if

$$\alpha_2^*(\alpha_1) < \hat{\alpha}_2(\theta) = \frac{(2-\theta)(1-2\theta)}{2(2-\theta)(1-\theta)-1}$$

Using the expression of α_2^* in Lemma 1 in the text, we obtain that $\alpha_2^*(\alpha_1) < \hat{\alpha}_2(\theta)$ if and only if

$$\alpha_2^{max}(\theta, \alpha_1)\left(1 + \sqrt{1 - \frac{C_p}{C_{max}(\theta, \alpha_1)}}\right) < \hat{\alpha}_2(\theta). \quad (3.1)$$

That is, if and only if

$$\sqrt{1 - \frac{C_p}{C_{max}(\theta, \alpha_1)}} < \frac{\hat{\alpha}_2(\theta)}{\alpha_2^{max}(\theta, \alpha_1)} - 1. \quad (3.2)$$

For this inequality to be verified, a necessary condition is that the right hand side is positive. That is, we must have:

$$\hat{\alpha}_2(\theta) > \alpha_2^{max}(\theta,\alpha_1) = \frac{1}{2}\frac{1-(2\theta-1)\alpha_1}{\frac{1}{2-\theta}+\left(2(1-\theta)-\frac{1}{2-\theta}\right)\alpha_1}\left(=\frac{(2-\theta)(1-(2\theta-1)\alpha_1)}{2(1+(2(2-\theta)(1-\theta)-1)\,\alpha_1)}\right). \tag{3.3}$$

Observe that $\alpha_2^{max}(\theta,\alpha_1)$ decreases with α_1. Moreover, $\alpha_2^{max}(\theta,0) = (2-\theta)/2$ and $\alpha_2^{max}(\theta,1) = 1/2$. We have:

$$\begin{aligned}\alpha_2^{max}(\theta,0)-\hat{\alpha}_2(\theta) &= \frac{2-\theta}{2}-\frac{1-2\theta}{2(1-\theta)-\frac{1}{2-\theta}} = \frac{2(1-\theta)(2-\theta)-1-2(1-2\theta)}{2\left(2(1-\theta)-\frac{1}{2-\theta}\right)}\\ &= \frac{2(1-\theta)(2-\theta)-4(1-\theta)+1}{2\left(2(1-\theta)-\frac{1}{2-\theta}\right)} = \frac{-2\theta(1-\theta)+1}{2\left(2(1-\theta)-\frac{1}{2-\theta}\right)} > 0,\end{aligned}$$

for $\theta \le 1/2$. Moreover,

$$\begin{aligned}\alpha_2^{max}(\theta,1)-\hat{\alpha}_2(\theta) &= \frac{1}{2}-\frac{1-2\theta}{2(1-\theta)-\frac{1}{2-\theta}} = \frac{1-\theta-\frac{1}{2(2-\theta)}-1+2\theta}{2(1-\theta)-\frac{1}{2-\theta}}\\ &= \frac{\theta-\frac{1}{2(2-\theta)}}{2(1-\theta)-\frac{1}{2-\theta}} = \frac{\theta(2-\theta)-\frac{1}{2}}{2(1-\theta)(2-\theta)-1}\\ &= \frac{-(1-\theta)^2+\frac{1}{2}}{2(1-\theta)(2-\theta)-1}.\end{aligned}$$

Therefore $\alpha_2^{max}(\theta,1)-\hat{\alpha}_2(\theta) > 0$ iff $\theta > \frac{\sqrt{2}-1}{\sqrt{2}}$. Consequently if $\theta > \frac{\sqrt{2}-1}{\sqrt{2}}$ then $\alpha_2^{max}(\theta,\alpha_1)-\hat{\alpha}_2(\theta) > 0$ for all α_1 and Condition (3.3) cannot hold true. Hence, the derivative of α_2^* with respect to α_1^* cannot be positive if $\theta > \frac{\sqrt{2}-1}{\sqrt{2}}$..

If $\theta < \frac{\sqrt{2}-1}{\sqrt{2}}$, there exists $\bar{\alpha}_1(\theta) \in [0,1]$ such that $\alpha_2^{max}(\theta,\bar{\alpha}_1(\theta)) = \hat{\alpha}_2(\theta)$ and for all $\alpha_1 > \bar{\alpha}_1(\theta)$, $\alpha_2^{max}(\theta,\alpha_1) < \hat{\alpha}_2(\theta)$, since $\alpha_2^{max}(\theta,\alpha_1)$ is decreasing with α_1,. Solving $\alpha_2^{max}(\theta,\bar{\alpha}_1(\theta)) = \hat{\alpha}_2(\theta)$ for $\bar{\alpha}_1(\theta)$, we obtain:

$$\bar{\alpha}_1(\theta) = \frac{1}{\left(1-\frac{\theta}{1-\theta}\right)\left[2(2-\theta)-\frac{1}{1-\theta}\right]}+\frac{\theta^2}{(1-2\theta)[2(1-\theta)(2-\theta)-1]}. \tag{3.4}$$

We deduce that for $\theta < \frac{\sqrt{2}-1}{\sqrt{2}}$, $\bar{\alpha}_1(\theta)$ increases with θ. Moreover,

$$\bar{\alpha}_1(0) = \frac{1}{3}$$

and

$$\bar{\alpha}_1(\frac{\sqrt{2}-1}{\sqrt{2}}) = \frac{\frac{1}{2} + \left(1 - \frac{1}{\sqrt{2}}\right)^2}{(\sqrt{2}-1)\left[2\frac{1}{\sqrt{2}}\left(1+\frac{1}{\sqrt{2}}\right)-1\right]} = 1.$$

Thus, if $\theta < \frac{\sqrt{2}-1}{\sqrt{2}}$ and $\alpha_1 > \bar{\alpha}_1(\theta)$, Condition (3.2) can be satisfied. This condition is equivalent to:

$$C_p > \bar{C}_p(\theta), \tag{3.5}$$

where

$$\bar{C}_p(\theta) \equiv \frac{C_{max}(\theta,\alpha_1)}{\alpha_2^{max}(\theta,\alpha_1)^2}(2\alpha_2^{max}(\theta,\alpha_1) - \hat{\alpha}_2(\theta))\hat{\alpha}_2(\theta)$$

We have

$$\frac{C_{max}(\theta,\alpha_1)}{\alpha_2^{max}(\theta,\alpha_1)^2} = \frac{\theta}{2}\left[\frac{1}{2-\theta} + \left(2(1-\theta) - \frac{1}{2-\theta}\right)\alpha_1\right].$$

Moreover:

$$\begin{aligned}2\alpha_2^{max}(\theta,\alpha_1) - \hat{\alpha}_2(\theta) &= \frac{1-(2\theta-1)\alpha_1}{\frac{1}{2-\theta} + \left(2(1-\theta)-\frac{1}{2-\theta}\right)\alpha_1} - \frac{1-2\theta}{2(1-\theta)-\frac{1}{2-\theta}} \\ &= \frac{\frac{2(1-\theta)^2}{2-\theta}}{\left(2(1-\theta)-\frac{1}{2-\theta}\right)\left[\frac{1}{2-\theta} + \left(2(1-\theta)-\frac{1}{2-\theta}\right)\alpha_1\right]}\end{aligned}$$

Thus:

$$\bar{C}_p(\theta) = \frac{\theta(1-\theta)^2}{2-\theta}\frac{\hat{\alpha}_2(\theta)}{2(1-\theta)-\frac{1}{2-\theta}} = \frac{\theta(1-\theta)^2}{2-\theta}\frac{1-2\theta}{\left(2(1-\theta)-\frac{1}{2-\theta}\right)^2} = \frac{\theta(1-\theta)^2(2-\theta)(1-2\theta)}{(2(1-\theta)(2-\theta)-1)^2}.$$

Q.E.D

Finally, we observe in equilibrium, the condition $\alpha_1^*(C_r) > \bar{\alpha}_1(\theta)$ is equivalent to:

$$C_r < \frac{\theta}{2}\left(\frac{1}{4} - \max\left(\bar{\alpha}_1(\theta) - \frac{1}{2}, 0\right)^2\right) = \bar{C}_r(\theta).$$

4 Proof that C_{max} decreases with α_1 when $\theta > \frac{\sqrt{2}-1}{\sqrt{2}}$.

For $\theta > 1/2$, we can write C_{max} as

$$C_{max}(\theta, \alpha_1) = \frac{\theta}{4}\bar{\alpha}_2(\theta, \alpha_1)(1 - (2\theta - 1)\alpha_1),$$

which is the product of two decreasing and positive functions of α_1.

For $\theta < 1/2$, we have shown in the previous section that

$$C_{max}(\theta, \alpha_1)\left(2 - \frac{\hat{\alpha}_2(\theta)}{\alpha_2^{max}(\theta, \alpha_1)}\right)\frac{\hat{\alpha}_2(\theta)}{\alpha_2^{max}(\theta, \alpha_1)} = \bar{C}_p(\theta) > 0. \tag{4.1}$$

Notice that the the function $h(x) = x(2 - x)$ reaches a maximum for $x = 1$, increases for $x < 1$ and decreases for $x > 1$. The ratio $r(\theta, \alpha_1) = \frac{\hat{\alpha}_2(\theta)}{\alpha_2^{max}(\theta, \alpha_1)}$ increases with α_1. For $\theta > \frac{\sqrt{2}-1}{\sqrt{2}}$, $r(\theta, \alpha_1)$ is always less than 1. Hence $r(\theta, \alpha_1)(2-r(\theta, \alpha_1))$ increases with α_1. We deduce that $C_{max}(\theta, \alpha_1)$ must decrease with α_1 for $\theta > \frac{\sqrt{2}-1}{\sqrt{2}}$ since $C_{max}(\theta, \alpha_1)r(\theta, \alpha_1)(2-r(\theta, \alpha_1))$ is strictly positive does not depend on α_1 (see eq.(4.1)).