Online Appendix to “Inventory Management, Dealers’ Connections, and Prices in OTC Markets”

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This Online Appendix consists in three parts. Part I discusses how the network structure assumed in our model compares to that observed in real-world OTC markets. Part II provides additional proofs, omitted in the main text for brevity. Part III develops several extensions and robustness checks of the model.
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Evidence on the Network Structure of OTC Markets

In this Appendix, we discuss how the network structure assumed in our model compares to that observed in real-world OTC markets. To this end, we draw on data from the European interbank market and the inter-dealer segment of the U.S. corporate bond market. We apply two different algorithms for the identification of core members. We show that peripheral dealers engage in bilateral transactions, and are heterogeneous in their connectedness to core dealers. Overall, these stylized facts are consistent with our modelling assumptions.

A The European interbank market

A.1 Empirical analysis

We obtain data on the euro area overnight unsecured interbank market (“the European interbank market”) from the European Central Bank. The dataset contains trade-level information on unsecured overnight loans contracted from payments data using a variant of the so-called Furfine (1999) algorithm. The sample period spans June 2008 - June 2012. A more detailed description of the data is provided in Frutos, Garcia de Andoain, Heider, and Papsdorf (2017). We consolidate trading activity at the group level using data from SWIFT and an internal database maintained at the ECB, and exclude intragroup transactions. Finally, we restrict the dataset to trades between banks residing in the euro area to ensure that counterparties are subject to reserve requirements and have access to the ECB’s standing

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1One of the authors (Peter Hoffmann) is a member of one of the user groups with access to TARGET2 data in accordance with Article 1(2) of Decision ECB/2010/9 of 29 July 2010 on access to and use of certain TARGET2 data. The ECB and the PSSC have checked the paper against the rules for guaranteeing the confidentiality of transaction-level data imposed by the PSSC pursuant to Article 1(4) of the above mentioned issue. The views expressed in the paper are solely those of the authors and do not necessarily represent the views of the Eurosystem.

2The data has been used in several other studies, e.g. Garcia de Andoain, Hoffmann, and Manganelli (2014); de Andoain, Heider, Hoerova, and Manganelli (2016).
facilities. This leaves us with a total of 456,599 transactions among 635 banks. Note that the interbank market can be considered as an inter-dealer market because banks ultimately trade to smoothe liquidity shocks arising from customer payments.³

For simplicity, we focus on binary networks where bilateral links are defined by the existence of at least one transaction. It is convenient to represent a network with N nodes via a binary adjacency matrix \( A \) such that \( a_{i,j} = 1 \) if there exists a link between node \( i \) and \( j \), and \( a_{i,j} = 0 \) otherwise. We also focus on undirected networks, such that \( A \) is symmetric. Economists and social scientists sometimes devote particular attention to so-called core-periphery networks as a theoretical benchmark. A network follows a core-periphery structure if the core forms a complete network and there are no bilateral links in the periphery. If one permutes row and columns so that the core region is collected on the upper left block, this implies the following structure

\[
A = \begin{bmatrix}
CC & CP \\
PC & PP
\end{bmatrix} = \begin{bmatrix}
1 & CP \\
PC & 0
\end{bmatrix}.
\]

Here, \( 1 \) denotes a hollow (zeros on the diagonal) \( N_c \times N_c \) matrix with all off-diagonal elements equal to one, \( 0 \) is an \( N_p \times N_p \) matrix of zeros, and \( N_c \) and \( N_p \) are the number of core and periphery members, respectively.

We start out with a graphical overview of the European interbank market network. Figure IB.1 plots the adjacency matrix, where rows/columns have been permuted so that lower indices correspond to banks with more connections. Black dots indicate the existence of a bilateral link. For improved readability, Panel A plots the upper left part of the matrix that contains 99% of all links. Panel B zooms in further and depicts the part containing 80% of all links. Overall, the European interbank market does not exhibit a core-periphery structure in the strict sense. While most links are among the first 150-200 banks, only a much smaller subset of around 30-50 banks appears to form a complete network. Accordingly, we observe

³Note that banks can typically borrow funds from the central bank at a penalty rate \( R^p \) in case they face a liquidity shortage that they cannot cover in the market. Similarly, excess liquidity can be deposited at the central bank at some deposit rate \( R^D \). These two rates can be interpreted as the linear inventory costs faced by dealers our model.
numerous bilateral links among banks that can be considered as part of the periphery. This is consistent with the market structure in our model, where peripheral dealers trade with each other.

In order to formalize our analysis, we apply two methodologies to separate banks into “core” and “periphery”. Both methods differ in the degree of stringency they impose on the “core”. The first one is the algorithm developed in Craig and von Peter (2014) (henceforth CvP), which minimizes the distance of the observed network to the idealized core-periphery network from equation (I.A.1).4 One issue with the CvP algorithm is that it takes a network strict core-periphery as null hypothesis and therefore group dealers so as to be the closest possible to this hypothesis. However, the underlying network may have a different structure, which can render alternative methods of defining a set of core banks more appropriate. For this reason, we use a second method that is more agnostic on the actual structure of the market. It defines the core as the set of all banks belonging to the largest complete subgraph (“clique”). If the largest clique is not unique, we choose the one with the largest trading volume.5 This methodology is more restrictive on the core because it requires a strictly complete network. At the same time, it places no restriction on the periphery. It can be viewed as a special case of the CvP algorithm when errors in the core (periphery) receive infinite (zero) weights.

[Insert Figure IB.2 here.]

We apply the two algorithms to monthly networks in order to additionally shed light on potential time-series variation. Panel A of Figure IB.2 depicts the number of core banks over time. The core under the CvP algorithm fluctuates between 20-40 banks over the sample period. While it is largely stable from one month to the next, it exhibits two periods where the size of the core declines significantly. The first one is around the Lehman bankruptcy in September 2008, and the other one corresponds to the most intense period of the euro

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4CvP additionally require the block matrices CP and PC to be “row regular” and “column regular”, respectively, i.e., they require that each peripheral bank is linked to at least one core bank. Eisfeldt, Herskovic, Rajan, and Siriwardane (2018) apply a similar algorithm, where they require that all elements of CP and PC are equal to one, i.e., each peripheral bank is connected to all core banks. Applying their algorithm instead of CvP delivers qualitatively very similar results.

5Alternatively defining the core as the set of counterparties that belong to at least one of the largest cliques delivers qualitatively similar results.
crisis in the second half of 2011. While the maximum clique algorithm returns a considerably smaller core of 7-13 banks, its time-series pattern is highly correlated to the one from the CvP methodology (correlation coefficient 0.75). Panel B plots the error score from the CvP algorithm over time (solid line), which is defined as the sum of links that are at odds with the structure in equation I.A.1 relative to the number of links. The error score oscillates between 45% and 60%, suggesting that the structure of the European interbank market is in fact not very close to a strict core-periphery network. Note that there are two types of errors: existing periphery-periphery (P2P) links, and missing core-core (C2C) links. Panel B of Figure IB.2 additionally plots the error score due to P2P links (dashed line), which fluctuates between 30% and 50%. This suggests that up to half of all existing bilateral links in the European interbank market are among peripheral banks.

Figure IB.3 provides a decomposition of turnover in the interbank market into three different types of matches: Core-core (solid line), core-periphery (C2P, dashed line), and periphery-periphery (dash-dot line). Panel A shows that, under the CvP methodology, turnover due to P2P transactions accounts for between 20% and 60% of total turnover, and is thus economically very significant. Interestingly, P2P volume increases strongly in crisis periods, i.e., in times when banks struggle to find counterparties that are willing to trade. In our model, such periods could be interpreted as an increase in $\lambda$. C2C and C2P turnover fluctuate in the range 10-30% and 30-60%, respectively. Panel B reveals that P2P turnover accounts for up to 90% under the maximal clique algorithm, with a similar time-series pattern than under CvP. To summarize, this decomposition provides further evidence that the structure of the European interbank market deviates significantly from a perfect core-periphery network, and is consistent with our assumptions. In particular, bilateral trade among peripheral banks accounts for a substantial fraction of overall market activity, sometimes in excess of 50%.

One key assumption in our model is that peripheral dealers’ access to the core is heterogeneous. To shed light on the real-world relevance of this assumption, Figure IB.4 illustrates that this assumption is indeed quite realistic. It depicts a histogram of the number of core connections across peripheral banks.\(^6\) While a large fraction of peripheral banks have only

\(^6\)To construct this histogram, we record the relative frequency of core connections for peripheral banks.
few connections, some are connected to more than 10 core banks. Interestingly, around 30% of all peripheral banks never trade with a core bank. Figure IB.5 plots the time-series evolution of the fraction of active banks with zero and one connections to the core. These banks account for between one and two thirds of all active banks in a given month. Consistent with our interpretation of \( \lambda \), we observe a higher fraction of unconnected banks in crisis periods.

[Insert Figures IB.4, IB.5, and IB.6 here.]

Finally, we explore the relationship between trade size and connectedness. Figure IB.6 shows a box plot for the trade size of peripheral banks with different numbers of core dealers. As can be seen, there is no clear relationship between the two in the data. Accordingly, trade size is not a sufficient characteristic to infer banks’ connectedness. This vindicates our assumption that there is uncertainty on the connectedness of a particular counterparty.

A.2 Related empirical work on the network structure of interbank markets

While a thorough overview of the literature is beyond the scope of this section, we want to mention a few other papers that describe features of the network structure of interbank markets. Most papers show that these networks tend to be sparse, with a small number of active and well connected banks and a large number of less well connected and less active banks. Early examples include Furfine (1999) for the Fed Funds market and De Masi, Iori, and Caldarelli (2006) for the Italian interbank market. Langfield and Soramäki (2016) contains a more complete list of references. While there appears to be a rather widespread perception that these networks constitute core-periphery structures, this concept was only taken to the data more formally by Craig and von Peter (2014). The evidence is mixed, suggesting that not all interbank networks exhibit the same structure. While Craig and von Peter (2014) find error score of around 10% for the network of German interbank exposures, Anand, van Lelyveld, Banai, Friedrich, Garratt, Halaj, Fique, Hansen, Jaramillo, Lee, et al. (2018) find error scores ranging from less than 5% to more than 50% for a set of 13 interbank networks around the globe.
B  The U.S. corporate bond market

B.1  Empirical analysis

We obtained the academic TRACE dataset supplied by FINRA covering the U.S. corporate bond market from July 2002 to December 2015.\textsuperscript{7} The dataset covers virtually all USD-denominated corporate bond transactions. Compared to the ordinary TRACE dataset, the academic version contains anonymous IDs for FINRA-registered firms.\textsuperscript{8} Information on bond characteristics is obtained from Thomson Reuters. We restrict our sample to all USD-denominated corporate bonds, debentures and notes and further exclude bonds, which are extendible, putable, pay-in-kind, convertible, unrated or with an issue size below 10 million. In total, our dataset encompasses 33,474 corporate bonds. We correct for error reports in the TRACE dataset with a filter similar to the one proposed by Dick-Nielsen (2014). We also assign the correct IDs for give-up and locked-in trades.\textsuperscript{9} Furthermore, we delete transactions conducted under special conditions, when-issued trades, duplicate entries of inter-dealer trades and trades with the same reporting and counterparty ID. Finally, we restrict the sample to inter-dealer trades, which we define as transactions where both participants are FINRA registered firms. This leaves us with a total of 31.6 million among 3,492 dealers.

We follow the same steps as in the analysis of the interbank market. Figure CB.1 plots the binary adjacency matrix, where a bilateral link exists between counterparties $i$ and $j$ if they have traded at least once. Overall, the network structure of the U.S. corporate bond market is closer to a stylized core-periphery network than the European interbank market. However, in line with our model’s assumptions, we still observe a significant number of bilateral links in the periphery. This is particularly evident when looking at the zoomed-in version in Panel B.

[Insert Figures CB.1 and CB.2 here.]

Figure CB.2 reports the results from the application of the CvP and clique algorithms

\begin{itemize}
\item \textsuperscript{7}This dataset has been used in other studies, e.g., Friewald and Nagler (2019) and Bessembinder, Maxwell, Jacobsen, and Venkataraman (2018).
\item \textsuperscript{8}A list of the reporting firms is available here: https://www.finra.org/about/firms-we-regulate.
\item \textsuperscript{9}Give-up and locked-in trades are trades where a third-party (e.g., an inter-dealer broker or a trading platform) reports a trade conducted by one of its correspondent firms.
\end{itemize}
to detect the core. We use quarterly networks in order to have roughly the same number of observations for both markets.\textsuperscript{10} Under CvP, the core contains between 75 and 105 dealers (Panel A). While the core size is remarkably stable over time, we observe a permanent increase in 2008, suggesting an increase in the core around the time of the default of Lehman Brothers. Note that this is the opposite pattern than for the interbank market. The discrepancy may be due to the absence of counterparty risk in the corporate bond market. While the core size is smaller under the clique algorithm at 25-45 dealers, this corresponds more closely to the numbers reported in other empirical studies, e.g., Di Maggio, Kermani, and Song (2017) or Bessembinder, Spatt, and Venkataraman (2019). In any case, the time-series pattern is again rather similar. Panel B depicts the error score (solid line) from the CvP algorithm over time. It ranges from 13\% to 23\%, confirming the impression from the visualization of the adjacency matrix that the structure of the U.S. corporate bond market is closer to that of a stylized core-periphery network. However, even in this case between 10\% and 15\% of all links are among peripheral banks (dashed line).

Figure CB.3 provides the decomposition of turnover into C2C (solid line), C2P (dashed line), and P2P (dash-dot line) matches. Under the CvP methodology, turnover due to P2P transactions fluctuates between 2\% and 7\% of total turnover. However, this amounts to between 4\% and 18\% of total turnover by peripheral dealers, suggesting that P2P trading is economically important for them. The picture changes substantially when applying the clique algorithm. In this case, P2P turnover accounts for 20-75\% of total turnover. This shows that the magnitude of bilateral trading activity among peripheral dealers is quite sensitive to the methodology used to identify core dealers. Our modelling assumptions are more in line with a strict definition of core dealers.

Next, we examine heterogeneity across peripheral dealers’ access to core dealers. Figures CB.4 and CB.5 show that our key assumption on dealer heterogeneity maps to the real world. A large fraction of peripheral dealers have only few connections to the core, while others have access to a large number of core dealers. Similar to the interbank market, around 30\% of

\textsuperscript{10}Using monthly bond market networks delivers qualitatively very similar results.
all dealers never trade with a core dealer under the CvP algorithm. Figure CB.5 shows that around one third of active dealers in a given month have zero or one connection to the core. This figure varies little over time, suggesting that shocks to $\lambda$ are less common than in the interbank market. While the share of unconnected dealers is almost twice as high under the clique algorithm, it also varies little over time.

[Insert Figures CB.4 and CB.5 here.]

Finally, Figure CB.6 illustrates the relationship between trade size and peripheral dealers connectedness. The results using the CvP are very similar to the interbank market, i.e. more connected dealers do not trade larger tickets. This is particularly true for the median trade, whose size is essentially flat across different levels of connectedness. Only for the trades at the 90th percentile we observe somewhat larger trades for more connected peripheral dealers.

[Insert Figure CB.6 here.]

B.2 Related empirical work on the network structure of fixed-income markets

The increasing availability of transaction-level data from OTC markets has spurred new empirical research, in particular on fixed-income markets. See Bessembinder, Spatt, and Venkataraman (2019) for a survey. The observed networks for these markets share important features of a stylized core-periphery structure, e.g. the concentration of activity among a small set of large counterparties. However, there has been, to our knowledge, little formal analysis. Different papers use different methods for the identification of core dealers. Li and Schürhoff (2019) report a set of 10 to 30 core dealers in the U.S. municipal bond market, but do not provide details about their identification. Bessembinder, Maxwell, Jacobsen, and Venkataraman (2018) single out a set of 10 to 12 core dealers that capture 70% of market share in the U.S. corporate bond market. The analysis in Di Maggio, Kermani, and Song (2017) defines the core are the group of the 50 largest dealers in the same market. Finally, 

\footnote{One exception is the paper by Eisfeldt, Herskovic, Rajan, and Siriwardane (2018), who examine the structure of the CDS market using a similar algorithm than CvP.}
Hollifield, Neklyudov, and Spatt (2017) identify core dealers as those in the top 5% of the eigenvector centrality distribution. It is important to note that these papers simply aim to highlight that observed networks are not random, for which their methods are useful and the particular choice of core dealers is less important than in our paper.

C Figures

Figure IB.1: Visualization of adjacency matrix - Interbank market

Panel A: Full Network (99% of all links)  
Panel B: Zoom (80% of all links)

This Figure visualizes the binary adjacency matrix for the European interbank market, where black dots represent bilateral links. Rows/columns have been permuted so that lower indices correspond to counterparties with more connections. Panel A plots the upper left submatrix containing 99% of all links, while Panel B plots the upper left submatrix containing 80% of all links.
Figure IB.2: Core Size - Interbank market

A: Core size

B: CvP error score

Panel A of this Figure depicts the estimated core size for the European interbank market over time. The core size is determined using two methods: i) the CvP algorithm and ii) the maximum clique that accounts for the largest trading volume. Panel B plots the error score from the CvP algorithm over time, defined as the number of errors divided by the number of links.

Figure IB.3: Turnover share of different types of matches

A: CvP

B: Clique

This Figure depicts the evolution of turnover in the interbank market for three different types of matches: Core-core (C2C, bold line), core-periphery (C2P, dashed line), and periphery-periphery (P2P, dash-dot line). In Panel A, banks are assigned to core and periphery using the CvP algorithm, while Panel B uses the clique algorithm.
Figure IB.4: Heterogeneity in core connections

A: CvP

B: Clique

This Figure presents a histogram of the fraction of peripheral dealers with connections with different numbers of core dealers. In Panel A, banks are assigned to core and periphery using the CvP algorithm, while Panel B uses the clique algorithm.

Figure IB.5: Heterogeneity in core connections over time

A: CvP

B: Clique

This Figure plots the number of peripheral dealers with zero, one, or more than one connections to core dealers over time. In Panel A, banks are assigned to core and periphery using the CvP algorithm, while Panel B uses the clique algorithm.
This Figure provides a box plot for the trade size for different categories of peripheral dealers depending on their number of connections to core dealers. Whiskers indicate the 10th and 90th percentiles, while the boxes span the 25th to 75th percentiles, with the horizontal line inside the box indicating the median. In Panel A, banks are assigned to core and periphery using the CvP algorithm, while Panel B uses the clique algorithm.
Figure CB.1: Visualization of adjacency matrix - TRACE

Panel A: Full Network (99% of all links)  Panel B: Zoom (80% of all links)

This Figure visualizes the binary adjacency matrix for the U.S. corporate bond market, where black dots represent bilateral links. Rows/columns have been permuted so that lower indices correspond to counterparties with more connections. Panel A plots the upper left submatrix containing 99% of all links, while Panel B plots the upper left submatrix containing 80% of all links.

Figure CB.2: Core Size - TRACE

A: Core size  B: CvP error score

Panel A of this Figure depicts the estimated core size for the U.S. corporate bond market over time. The core size is determined using two methods: i) the CvP algorithm and ii) the maximum clique that accounts for the largest trading volume. Panel B plots the error score from the CvP algorithm over time, defined as the number of errors divided by the number of links.
This Figure depicts the evolution of turnover in the interbank market for three different types of matches: Core-core (C2C, bold line), core-periphery (C2P, dashed line), and periphery-periphery (P2P, dash-dot line). In Panel A, banks are assigned to core and periphery using the CvP algorithm, while Panel B uses the clique algorithm.

This Figure presents a histogram of the fraction of peripheral dealers with connections with different numbers of core dealers. In Panel A, banks are assigned to core and periphery using the CvP algorithm, while Panel B uses the clique algorithm.
This Figure plots the number of peripheral dealers with zero, one, or more than one connections to core dealers over time. In Panel A, banks are assigned to core and periphery using the CvP algorithm, while Panel B uses the clique algorithm.

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Part II

Additional Proofs

This Appendix gives additional proofs not reported in the main text for brevity. Section A shows how to derive mixed strategy equilibria of our model when they exist. Section B gives the stationary probabilities of each possible event in the peripheral market for a given strategy profile of peripheral dealers. Section C provides the proof of Lemma 4, Section D completes the proof of Proposition 4, Section E proves that $W(\Sigma^*)$ increases in $\lambda$, Section F details how we simulated the model to produce the data used in Section 5.2, and Section G gives the proofs of Implications 1 to 5.

A Mixed Equilibria

Proposition 1 gives the equilibrium in pure strategy for given values of the exogenous parameters ($\lambda, \alpha^{pe}, \kappa$). Moreover, if such an equilibrium exists, it is unique. However, as explained after the proposition, there exist parameter values for which no pure strategy equilibrium exists when $\kappa$ is finite. In this section, we explain how one can characterize mixed strategy equilibria in this case.

We focus on the case $\alpha^{pe} > 1/2$ (the case $\alpha^{pe} < 1/2$ is symmetric). In this case, Lemma 3 implies that a mixed strategy obtains if and only if $p^{co} = (1 - \omega_b)C^b - \omega_bC^s$. Consider a mixed strategy equilibrium with $\Sigma = (1, \theta_b, \gamma_s, 1)$, with $\theta_b \in (0, 1)$ and $\gamma_s \in (0, 1)$. Such an equilibrium requires that (i) peripheral sellers are indifferent between making an offer or trading directly with core dealers, and that (ii) unconnected peripheral buyers are indifferent between making an offer accepted by all peripheral sellers and offer accepted only by unconnected peripheral sellers. Using the analysis of the ACD regime in the proof of Lemma 3, it is easily shown that having $p^{co} = (1 - \omega_b)C^b - \omega_bC^s$ is a sufficient condition for requirements (i) and (ii).

Now, to close the construction of the mixed strategy equilibrium, we show that one can find $\theta_b$ and $\gamma_s$ such that $p^{cos}$ is equal to $(1 - \omega_b)C^b - \omega_bC^s$ when $\alpha^{co} \in (\alpha^{ICS}, \alpha^{ACD})$ (i.e.,
precisely when no pure strategy equilibrium exists; see Proposition 1). Using (13), this condition is equivalent to finding \( \theta_b \) and \( \gamma_s \) solving:

\[
\Delta^*(\alpha^{pe}, \lambda, (1, \theta_b, \gamma_s, 1)) = -z_0^{co} - \kappa \Phi^{-1}(1 - \omega_b).
\] (II.A.1)

The right hand side of this equation is independent of \( \theta_b \) and \( \gamma_s \). Thus, if (i) \( \Delta^*(\alpha^{pe}, \lambda, (1, \theta_b, \gamma_s, 1)) \) decreases in \( \theta_b \) and \( \gamma_s \) and (ii)

\[
\Delta^*(\alpha^{pe}, \lambda, (1, 1, 1, 1)) \leq -z_0^{co} - \kappa \Phi^{-1}(1 - \omega_b) \leq \Delta^*(\alpha^{pe}, \lambda, (1, 0, 0, 1)),
\] (II.A.2)

then (II.A.1) has at least one solution.

We first show that \( \Delta^*(\alpha^{pe}, \lambda, (1, \theta_b, \gamma_s, 1)) \) decreases in \( \theta_b \) and \( \gamma_s \) using the analytical expression for \( \Delta^*(\alpha^{pe}, \lambda, (1, \theta_b, \gamma_s, 1)) \) given by (II.B.2) (in Section B). Using this expression, we find that the derivative of \( \Delta^* \) with respect to \( \theta_b \) has the sign of the following product:

\[
[-1 + \gamma_s - \gamma_s(1 - \lambda)\lambda \pi_b][1 - \lambda \pi_b + \gamma \lambda \pi_b(\lambda \pi_b(2 - \lambda(2 - \lambda)) - 1)].
\] (II.A.3)

The first term is negative. As \( 2 - \lambda(2 - \lambda) \geq 0 \), the second term is greater than \( 1 - \lambda \pi_b - \gamma \lambda \pi_b \geq 1 - 2\lambda \pi_b \). This quantity is greater than 0 because \( \lambda \leq 1/2 \). Hence, the product in (II.A.3) is negative, so that \( \Delta^* \) decreases in \( \theta_b \).

Similarly, the derivative of \( \Delta^* \) with respect to \( \gamma_s \) has the sign of:

\[
[-1 + \lambda \pi_b(1 + \theta_b(1 - \lambda))][1 - \lambda \pi_b(2 - \lambda(2 - \lambda)) - \theta_b(1 - \lambda)(1 - \lambda \pi_b(1 - \lambda))].
\] (II.A.4)

The first term of this product is negative. The second term is decreasing in \( \theta_b \) and is thus greater than the value when \( \theta_b = 1 \), which reduces to \( \lambda(1 - \pi_b) \). Thus, the second term is positive and the product (II.A.4) is negative. It follows that \( \Delta^* \) decreases in \( \gamma_s \).

Now we show that condition (II.A.2) is equivalent to \( \alpha^{co} \in (\alpha^{-ICS}, \alpha^{-ACD}) \). To see this, observe that (II.A.2) is equivalent to

\[
-\frac{\Delta^*(\alpha^{pe}, \lambda, (1, 0, 0, 1))}{\kappa} - \Phi^{-1}(1 - \omega_b) \leq (2\alpha^{co} - 1) \leq -\frac{\Delta^*(\alpha^{pe}, \lambda, (1, 1, 1, 1))}{\kappa} - \Phi^{-1}(1 - \omega_b).
\] (II.A.5)
As $\Sigma^{ACD} = (1, 1, 1, 1)$ and $\Sigma^{ICS} = (1, 0, 0, 1)$, and using the definition of $\alpha^{co} \in (\alpha^{ICS}, \alpha^{ACD})$ in the proof of Proposition 1, we deduce that Condition (II.A.5) is equivalent to $\alpha^{co} \in (\alpha^{ICS}, \alpha^{ACD})$. Thus, we have characterized full equilibria in mixed strategies precisely in the region in which a full equilibrium in pure strategies does not exist.

B Stationary Probabilities and Derivation of $\Delta^*(\alpha^{pe}, \lambda; \Sigma)$

Each time a new peripheral dealer is chosen to make a decision, eight possible events can occur: (i) A connected seller makes an offer or goes to the core market without making an offer; (ii) A connected seller accepts an offer; (iii) An unconnected seller makes an offer; (iv) An unconnected seller accepts an offer; (v) A connected buyer makes an offer or goes to the core market without making an offer; (vi) A connected buyer accepts an offer; (vii) An unconnected buyer makes an offer; (viii) An unconnected buyer accepts an offer.

Let $M = (\pi_{i,j})$ be the transition matrix from event $i \in \{1, 2, 3, 4, 5, 6, 7, 8\}$ to event $j \in \{1, 2, 3, 4, 5, 6, 7, 8\}$, where $\pi_{i,j}$ is the likelihood that event $i$ is followed by event $j$ given peripheral dealers’ strategy profile. We denote by $\mu_i$ the resulting stationary probabilities of event $i \in \{1, 2, 3, 4, 5, 6, 7, 8\}$ for a given pure strategy profile $\Sigma \in \{\Sigma^{ACD}, \Sigma^{ICS}\}$. Let $\mu$ be the (line) vector of the $\mu_i$s. It solves

$$\mu = \mu M \text{ and } 1(\mu)^t = 1, \quad (II.B.1)$$

where $1 = (1, 1, \ldots, 1)$ and $(\mu)^t$ is the transpose of $\mu$.

First, consider the case $\alpha^{pe} \geq \frac{1}{2}$. In this case, the only possible pure strategy profiles in equilibrium are $\Sigma^{ACD}$ and $\Sigma^{ICS}$ (see Lemma 3). Thus, $\theta_s = \gamma_b = 1$. It follows that:

$$M = \begin{pmatrix}
A(\pi_b, \lambda) & C(\pi_b, \lambda, 1, \gamma_s) \\
C(1, \lambda, \theta_b, 1) & A(1, \lambda)
\end{pmatrix},$$
where \( A(x, \lambda) \) and \( C(x, \lambda, \theta, \gamma) \) are two matrices defined as:

\[
A(x, \lambda) = (1-x) \begin{pmatrix}
(1-\lambda) & 0 & \lambda & 0 \\
(1-\lambda) & 0 & \lambda & 0 \\
(1-\lambda) & 0 & \lambda & 0 \\
(1-\lambda) & 0 & \lambda & 0 \\
\end{pmatrix},
\]

and

\[
C(x, \lambda, \theta, \gamma) = x \begin{pmatrix}
(1-\lambda) & 0 & \lambda(1-\gamma) & \lambda \gamma \\
(1-\lambda) & 0 & \lambda & 0 \\
(1-\lambda)(1-\theta) & (1-\lambda) \theta & 0 & \lambda \\
(1-\lambda) & 0 & \lambda & 0 \\
\end{pmatrix}.
\]

Define:

\[
\mu_{1a}(x, \lambda, \theta, \gamma) = (1-\lambda)[1-x\lambda(1+(1-\lambda)\theta)]/M(x, \lambda, \theta, \gamma),
\]

\[
\mu_{2a}(x, \lambda, \theta, \gamma) = x(1-\lambda)^2\lambda\theta[1-\gamma(1-x\lambda(1-\lambda))]/M(x, \lambda, \theta, \gamma),
\]

\[
\mu_{3a}(x, \lambda, \theta, \gamma) = \lambda[1-x+x\lambda(1-\lambda)\gamma(1-\theta(1-x(1-\lambda)))]/M(x, \lambda, \theta, \gamma),
\]

\[
\mu_{4a}(x, \lambda, \theta, \gamma) = x\lambda(1-\lambda)[1-\lambda\gamma(1+x\theta(1-\lambda)^2)]/M(x, \lambda, \theta, \gamma),
\]

\[
\mu_{1b}(x, \lambda, \theta, \gamma) = x(1-\lambda)^2[1-x\lambda\gamma(\theta+(1-\theta)\lambda)]/M(x, \lambda, \theta, \gamma),
\]

\[
\mu_{2b}(x, \lambda, \theta, \gamma) = x\lambda(1-\lambda)[1-x+x\lambda(1-\lambda)\gamma(1-\theta(1-x(1-\lambda)))]/M(x, \lambda, \theta, \gamma),
\]

\[
\mu_{3b}(x, \lambda, \theta, \gamma) = x\lambda(1-\lambda)[1-\gamma(1-x\lambda(1-\lambda))]M(x, \lambda, \theta, \gamma),
\]

\[
\mu_{4b}(x, \lambda, \theta, \gamma) = x\lambda[\lambda-x\lambda+(1-\lambda)\gamma(1-x\lambda\theta)(1-\lambda x(1-\lambda))]/M(x, \lambda, \theta, \gamma),
\]

\[
M(x, \lambda, \theta, \gamma) = (1+x)[1-x\lambda(1+\theta\gamma(1-\lambda)(1-\lambda x(1-\lambda)))].
\]

Solving (II.B.1), we obtain the stationary measures. When \( \alpha^{pe} \geq \frac{1}{2} \), we have \( \mu_i = \mu_{ia}(\pi_b, \lambda, \theta_b, \gamma_b) \) for \( i \leq 4 \) and \( \mu_i = \mu_{i-4,b}(\pi_b, \lambda, \theta_b, \gamma_b) \) for \( i > 4 \). For \( \alpha^{pe} < \frac{1}{2} \), a symmetric reasoning yields \( \mu_i = \mu_{ib}(\pi_s, \lambda, \theta_s, \gamma_b) \) for \( i \leq 4 \) and \( \mu_i = \mu_{i-4,a}(\pi_s, \lambda, \theta_s, \gamma_b) \) for \( i > 4 \).

**Derivation of \( \Delta^*(\alpha^{pe}, \lambda, \Sigma) \).** Suppose first that \( \alpha^{pe} \geq \frac{1}{2} \). We first compute the mass of connected sellers who trade with a core dealer. A connected seller trades with a core dealer when (i) she directly contacts a core dealer upon arrival or (ii) she makes an offer to a
peripheral dealer and this offer is rejected. The likelihood of the first event is $\mu_1(1-\gamma_s)$ while the likelihood of the second is $\mu_1\gamma_s ((1-\pi_b) + \pi_b(1-\lambda))$, where $\mu_1 = \mu_{1a}(\pi_b, \lambda, \theta_b, \gamma_s)$ (as shown above). Thus, the mass of connected sellers who eventually trade in the core market is: $\mu_1^{\text{core}} = \mu_1((1-\pi_b) + \pi_b(1-\lambda))\gamma_s + (1-\gamma_s)) = \mu_1(1-\pi_b\lambda\gamma_s)$. A similar reasoning for a connected buyer yields $\mu_5^{\text{core}} = \mu_5(1-\lambda)$ (accounting for the fact that $\pi_s = \gamma_b = 1$ when $\alpha^{pe} \geq \frac{1}{2}$) where $\mu_5 = \mu_{1b}(\pi_b, \lambda, \theta_b, \gamma_s)$. Thus, using the definition of $\Delta^*(\alpha^{pe}, \lambda, \Sigma)$ in (20) and the expressions for $\mu_1$ and $\mu_5$ when $\alpha^{pe} \geq 1/2$, we deduce that, for a given $\Sigma \in \{\Sigma^{ACD}, \Sigma^{ICS}\}$, we have:

$$\Delta^*(\alpha^{pe}, \lambda, \Sigma) = [\mu_1(1-\pi_b\lambda\gamma_s) - \mu_5(1-\lambda)] = \frac{N(\pi_b, \lambda, \theta_b, \gamma_s)}{M(\pi_b, \lambda, \theta_b, \gamma_s)}, \text{ for } \Sigma \in \{\Sigma^{ACD}, \Sigma^{ICS}\}$$

(II.B.2)

where

$$N(\pi, \lambda, \theta, \gamma) \equiv (1-\lambda)[1-\pi + \pi\lambda[(1-\lambda)(1-\theta) - \gamma]] + (1-\lambda)\lambda\gamma\pi^2[\theta + \lambda(1-\theta)(2-\lambda(2-\lambda))].$$

(II.B.3)

Now suppose that $\alpha^{pe} < \frac{1}{2}$. In this case, possible pure strategy profiles are $\Sigma \in \{\Sigma^{ACD}, \Sigma^{ICB}\}$ in equilibrium. By symmetry, one obtains that:

$$\Delta^*(\alpha^{pe}, \lambda, \Sigma^{ACD}) = -\Delta^*((1-\alpha^{pe}), \lambda, \Sigma^{ACD}) \text{ for } \alpha^{pe} < \frac{1}{2},$$

(II.B.4)

$$\Delta^*(\alpha^{pe}, \lambda, \Sigma^{ICB}) = -\Delta^*((1-\alpha^{pe}), \lambda, \Sigma^{ICS}) \text{ for } \alpha^{pe} < \frac{1}{2}.$$  (II.B.5)

C Proof of Lemma 4

The fact that $\Delta^*(\alpha^{pe}, \lambda, \Sigma^{ACD}) = -\Delta^*((1-\alpha^{pe}), \lambda, \Sigma^{ACD})$ for $\alpha^{pe} < \frac{1}{2}$ and $\Delta^*(\alpha^{pe}, \lambda, \Sigma^{ICB}) = -\Delta^*((1-\alpha^{pe}), \lambda, \Sigma^{ICS})$ for $\alpha^{pe} < \frac{1}{2}$ follows from (II.B.4) and (II.B.5). Thus, to prove points (i), (ii), and (iii), we just focus on the case $\alpha^{pe} \geq \frac{1}{2}$ since results are symmetric when $\alpha^{pe} < \frac{1}{2}$.

When $\alpha^{pe} \geq \frac{1}{2}$, there are two possible equilibrium outcomes in the peripheral market: $\Sigma^* = \Sigma^{ACD}$ or $\Sigma^* = \Sigma^{ICS}$. We prove points (i) and (ii) of Lemma 4 by considering these two cases separately. We then deduce point (iii). The proof uses the analytical expressions
of $\Delta^*(\alpha^{pe}, \lambda, \Sigma)$ derived in (II.B.2) in Appendix II. B.

**Points (i) and (ii).**

**Case 1:** $\Sigma^* = \Sigma^{ACD}$. In this case, $\theta_b = \gamma_s = 1$. Thus, using (II.B.2), we obtain:

$$\Delta^*(\alpha^{pe}, \lambda, \Sigma^{ACD}) = \frac{(1 - \lambda)(1 - \pi_b)(1 - \pi_b\lambda)}{(1 + \pi_b)(1 - \lambda\pi_b[2 - \lambda - \lambda\pi_b(1 - \lambda)^2])}. \quad (II.C.1)$$

In this case, one can rewrite $\Delta^*(\alpha^{pe}, \lambda, \Sigma^{ACD})$ as:

$$\Delta^*(\alpha^{pe}, \lambda, \Sigma^{ACD}) = \frac{(1 - \lambda)(1 - \pi_b\lambda)}{1 - \lambda\pi_b[2 - \lambda - \lambda\pi_b(1 - \lambda)^2]} \times z_0^{pe}, \quad (II.C.2)$$

since $\frac{1 - \pi_b}{1 + \pi_b} = 2\alpha^{pe} - 1 = z_0^{pe}$. Thus, differentiating (II.C.1) with respect to $\pi_b$, we obtain that $\frac{\partial \Delta^*}{\partial \pi_b}$ has the sign of $g(\pi_b, \lambda)$, with $g$ defined as:

$$g(\pi_b, \lambda) = -2 + \lambda[3 - \lambda(2 - \lambda) + 6\pi_b] - 2\lambda^2\pi_b(5 - \lambda(4 - \lambda(3 - \lambda)))$$

$$- \lambda(1 - \lambda)\pi_b^2[1 + \lambda(5 - \lambda(7 - \lambda(4 - \lambda)))] + 2\lambda^2(1 - \lambda)^3(1 + \lambda)p^3 - \lambda^3(1 - \lambda)^3\lambda^2p^4. \quad (II.C.3)$$

Differentiating thrice w.r.t. $\pi_b$, we obtain:

$$\frac{\partial g(\pi_b, \lambda)}{\partial \pi_b} = \lambda(6 - 2\pi_b) + 2\lambda^2[-5 + 4\lambda - 3\lambda^2 + \lambda^3], \quad (II.C.4)$$

$$\frac{\partial g(\pi_b, \lambda)}{\partial \pi_b^2} = -2\lambda - 2\lambda^2(2 - \lambda)(2 - \lambda(5 - \lambda(3 - \lambda))) + 12\lambda^2(1 - \lambda)^3\pi_b[1 + \lambda(1 - \pi_b)]. \quad (II.C.5)$$

$$\frac{\partial g(\pi_b, \lambda)}{\partial \pi_b^3} = 12(1 - \lambda)^3\lambda^2[1 + \lambda(1 - 2\pi_b)]. \quad (II.C.6)$$

As $\lambda \leq 1/2$, we have $\frac{\partial^3 g(\pi_b, \lambda)}{\partial \pi_b^3} \geq 0$. Hence:

$$\frac{\partial^2 g(\pi_b, \lambda)}{\partial \pi_b^2} \leq \frac{\partial^2 g(\pi_b, \lambda)}{\partial \pi_b^2}|_{\pi_b=1} = -2\lambda(1 - \lambda)[1 - \lambda + \lambda^2(4 - (1 + \lambda)^2)]. \quad (II.C.7)$$

This last quantity is negative for any $\lambda \leq 1/2$, so that $\frac{\partial^2 g(\pi_b, \lambda)}{\partial \pi_b^2} \leq 0$. Hence:

$$\frac{\partial g(\pi_b, \lambda)}{\partial \pi_b} \geq \frac{\partial g(\pi_b, \lambda)}{\partial \pi_b}|_{\pi_b=1} = 4(1 - \lambda)^3\lambda(1 + \lambda^2) > 0. \quad (II.C.8)$$
Hence \( g(\pi_b, \lambda) \leq g(1, \lambda) = -2(1 - \lambda)^4(1 + \lambda)^2 < 0 \). Consequently, \( \frac{\partial \Delta^*}{\partial \pi_b} \leq 0 \). As \( \pi_b = \frac{1 - \alpha^{pe}}{\alpha^{pe}} \), we deduce that \( \Delta^* \) increases in \( \alpha^{pe} \) as claimed in point (ii) of Lemma 4 when \( \Sigma^* = \Sigma^{ACD} \).

When \( \alpha^{pe} = 1/2 \), we have \( \pi_b = 1 \) and therefore \( \Delta^* = 0 \). As \( \Delta^* \) increases in \( \alpha^{pe} \), this shows that \( \Delta^* \) is positive for \( \alpha^{pe} > 1/2 \) and by symmetry negative for \( \alpha^{pe} < 1/2 \). This proves point (i) of Lemma 4 when \( \Sigma^* = \Sigma^{ACD} \).

**Case 2:** \( \Sigma^* = \Sigma^{ICS} \). In this case, \( \theta_b = \gamma_s = 0 \). Thus, using (II.B.2), we obtain:

\[
\Delta^*(\alpha^{pe}, \lambda, \Sigma^{ICS}) = \frac{(1 - \lambda)(1 - \pi_b(1 - \lambda(1 - \lambda)))}{(1 + \pi_b)(1 - \pi_b \lambda)}.
\]  

(II.C.9)

In this case, one can rewrite \( \Delta^*(\alpha^{pe}, \lambda, \Sigma^{ICS}) \) as:

\[
\Delta^*(\alpha^{pe}, \lambda, \Sigma^{ICS}) = \frac{(1 - \lambda)(1 - \pi_b(1 - \lambda(1 - \lambda)))}{(1 - \pi_b)(1 - \pi_b \lambda)} \times z_0^{pe}.
\]

(II.C.10)

since \( \frac{1 - \pi_b}{1 + \pi_b} = 2\alpha^{pe} - 1 = z_0^{pe} \).

To prove point (ii), we differentiate \( \Delta^* \) with respect to \( \pi_b \):

\[
\frac{\partial \Delta^*(\alpha^{pe}, \lambda, \Sigma^{ICS})}{\partial \pi_b} = -\frac{(1 - \lambda)[2 - \lambda(2 - \lambda + \pi_b(2 - \pi_b)) + \lambda(1 - \lambda)\pi_b^2]}{(1 + \pi_b)^2(1 - \lambda\pi_b)^2}.
\]

(II.C.11)

It is easy to show that \( \lambda(2 - \lambda + \pi_b(2 - \pi_b)) < 2 \) when \( \lambda \leq 1/2 \), so that \( \Delta^* \) decreases in \( \pi_b \) and thus increases in \( \alpha^{pe} \). As in the previous case, point (i) also follows. This proves points (i) and (ii) of Lemma 4 when \( \Sigma^* = \Sigma^{ICS} \).

**Point (iii).** We now compare the two cases. We have:

\[
\Delta^*(\alpha^{pe}, \lambda, \Sigma^{ICS}) - \Delta^*(\alpha^{pe}, \lambda, \Sigma^{ACD}) = \frac{\lambda\pi_b g(\pi_b, \lambda)}{(1 + \pi_b)(1 - \lambda\pi_b)[1 - \lambda\pi_b(2 - \lambda\pi_b)(1 - \lambda)])}.
\]

(II.C.12)

with \( g(\pi_b, \lambda) = 1 - \lambda - \lambda\pi_b[3(1 - \lambda\pi_b) + 7\lambda^2\pi_b(1 - \lambda) - \lambda(4 - \lambda)(1 - \pi_b\lambda^3)] \) (II.C.13)

Differentiating with respect to \( \pi_b \) twice, we obtain:

\[
\frac{\partial g(\pi_b, \lambda)}{\partial \pi_b} = -\lambda(1 - \lambda)[(3 - \lambda)(1 - 2\lambda^2\pi_b) - 2\lambda\pi_b(3 - 4\lambda)],
\]

(II.C.14)

\[
\frac{\partial^2 g(\pi_b, \lambda)}{\partial \pi_b^2} = 2\lambda^2(1 - \lambda)[3 - \lambda(4 - \lambda(3 - \lambda))].
\]

(II.C.15)
The last expression is positive, so that:

\[
\frac{\partial g(\pi_{b}, \lambda)}{\partial \pi_{b}} \leq \frac{\partial g(\pi_{b}, \lambda)}{\partial \pi_{b}}|_{\pi_{b}=1} = -\lambda(1 - \lambda)^2[3 - 2\lambda(2 - \lambda(2 - \lambda))].
\]  

(II.C.16)

This last quantity is negative, so that \( \frac{\partial g(\pi_{b}, \lambda)}{\partial \pi_{b}} \leq 0. \) Hence, \( g(\pi_{b}, \lambda) \geq g(1, \lambda) = (1 - \lambda)^4(1 + \lambda^2) > 0. \) This shows that \( \Delta^*(\alpha^{pe}, \lambda, \Sigma^{ICS}) > \Delta^*(\alpha^{pe}, \lambda, \Sigma^{ACD}). \)

D Proof of Proposition 4

Our goal in this section is twofold. First, we show that the equilibrium \( \Sigma^* \) of the peripheral market is inefficient when \( \alpha^{pe} \neq \frac{1}{2} \) (i.e., does not maximize \( W(\Sigma) \)). Second, we identify which strategy profiles are efficient (maximize \( W(\Sigma) \)). We only consider the case \( \alpha^{pe} < \frac{1}{2} \), as the analysis of the other case is identical.

For this analysis, we use several steps. First, we introduce the notion of matching plan, \( P \), for the social planner (Step 1), which is a more general notion than the notion of strategy profile, \( \Sigma \). Second, we compute aggregate gains from trade among peripheral dealers, \( W(P) \), for each possible matching plan \( P \) (Step 2). Third and fourth (Steps 3 and 4), we prove that the matching plans corresponding to the strategy profiles used in the ACD and ICB equilibrium regimes (i.e., those obtained when \( \alpha^{pe} < \frac{1}{2} \)) are inefficient. These steps prove the first part of Proposition 4. Then, in Step 5, we derive the efficient matching plan when \( \alpha^{pe} < \frac{1}{2} \) and show that it is implemented when peripheral dealers follow the strategy profile \( \Sigma^{FB^-} \). This step proves the second part of Proposition 4.

Step 1. Definition of a Matching Plan. Let \((k_1, j_1) \in \{U, C\} \times \{s, b\}\) and \((k_2, j_2) \in \{U, C\} \times \{s, b\}\) be a given pair of dealers, where \((k_2, j_2)\) is the successor of \((k_1, j_1)\) (i.e., the next dealer in the line of all dealers; see Section 3.2 in the paper). A matching plan specifies for each pair \((k_1, j_1)\) and \((k_2, j_2)\), the probability, \( p^{k_1j_1,k_2j_2} \), that the two dealers trade together. As there are 16 possible matches, a matching plan is a vector \( P_m \) with 16 entries. The social planner must choose \( P \) to maximize welfare (defined below). By definition, when \( j_1 = j_2 \) (two buyers or two sellers), we have \( p^{k_1j_1,k_2j_2} = 0 \). Thus, the social planner must ultimately choose only 8 of the \( p^{k_1j_1,k_2j_2} \). Moreover, if for a given \( k_1j_1 \) we have \( p^{k_1j_1,k_2j_2} = 0 \) for all values
of $k_2, j_2$ then a dealer with type $k_1, j_1$ is never matched with another peripheral dealer. If the dealer is connected, this means implicitly that the dealer is matched with a core dealer while if the dealer is unconnected this means that he will never trade.

As explained in the text, the central planner has the same information as that available to peripheral dealers when they make their decisions. Thus, every matching plan chosen by the central planner could be followed by dealers. We show below that, in general, the equilibrium strategy profile is not efficient in the sense that it does not correspond to the matching plan chosen by the social planner.

**Step 2. Calculation of aggregate gains from trade among peripheral dealers, $W(P)$**. We measure the efficiency of a given matching plan $P_m$ by the aggregate gains from trade per period, $W(P_m)$ among peripheral dealers. To compute $W(P_m)$, we proceed as follows. We first introduce a discount factor $\delta$. Then, for each pair of dealers of types $(k_1, j_1) \in \{U, C\} \times \{s, b\}$ and $(k_2, j_2) \in \{U, C\} \times \{s, b\}$, we denote by $W^{k_1, j_1, k_2, j_2}(P_m, \delta)$ the discounted total future gains from trade when the social planner follows the matching plan $P_m$, starting with a pair of dealers with types $(k_1, j_1)$ and $(k_2, j_2)$.

Let $\hat{W}^{k_1, j_1, k_2, j_2}(P_m, \delta) = (1 - \delta)W^{k_1, j_1, k_2, j_2}(P_m, \delta)$. The average gains from trade per period associated with the matching plan $P_m$ is then $W(P_m) = \lim_{\delta \to 1} \hat{W}^{k_1, j_1, k_2, j_2}(P_m, \delta)$. This welfare measure is always finite and does not depend on $(k_1, j_1, k_2, j_2)$ because as $\delta \to 1$ the initial pair is irrelevant for total welfare.

We first compute $W^{k_1, j_1, k_2, j_2}(P_m, \delta)$ for each of the 16 possible values for $k_1, j_1, k_2, j_2$. To this end, we define $L^{kj}(P_m, \delta)$ as the discounted future gains from trade when a dealer of type $kj$ is not matched with his predecessor and we express $W^{k_1, j_1, k_2, j_2}(P_m, \delta)$ as a function of $L^{k_2, j_2}(P_m, \delta)$ for the 16 possible values of $(k_1, j_1, k_2, j_2)$. We then express $L^{k_2, j_2}(P_m, \delta)$ as a function of $W^{k_2, j_2, k_3, j_3}(P_m, \delta)$ for $(k_3, j_3) \in \{U, C\} \times \{s, b\}$. This yields a system of recursive equations whose solution is $W^{k_1, j_1, k_2, j_2}(P_m, \delta)$. We omit the arguments $P_m$ and $\delta$ in the functions $W^{k_1, j_1, k_2, j_2}(P_m, \delta)$ and $L^{kj}(P_m, \delta)$ when there is no ambiguity.

**Cases 1 to 4:** In these cases, the first dealer is an unconnected buyer, i.e., $(k_1, j_1) = Ub$. Using the fact, that $p^{Ub, Ub} = p^{Ub, Cb} = 0$, we have:

$$W^{Ub, Ub} = W^{Ub, Cb} = L^{Ub}. \tag{II.D.1}$$
Moreover, as a seller is followed by a buyer with probability 1 (since \( \alpha_{pe} < 1/2 \)):

\[
W_{Ub,Us} = p_{Ub,Us} [C^s + C^b + \delta \lambda L^U + \delta(1 - \lambda) L^Cb] + (1 - p_{Ub,Us}) L^Us. \tag{II.D.2}
\]

and

\[
W_{Ub,Cs} = p_{Ub,Cs} [C^s + C^b + \delta \lambda L^U + \delta(1 - \lambda) L^Cb] + (1 - p_{Ub,Cs}) L^Cs. \tag{II.D.3}
\]

**Cases 5 to 8:** In these cases, the first dealer is an unconnected seller, i.e., \( k_1 j_1 = Us \). The analysis is almost identical to that of the previous cases, except that if the second dealer is a buyer (\( j_2 = b \)) and this buyer is not matched with the first dealer then the second dealer is followed by a seller with probability \( \pi_s \). We obtain:

\[
W_{Us,Ub} = p_{Us,Ub} [C^s + C^b + \delta(\lambda \pi_s L^Us + \lambda(1 - \pi_s) L^Ub + (1 - \lambda) \pi_s L^Cs + (1 - \lambda)(1 - \pi_s) L^Cb)] + (1 - p_{Us,Ub}) L^Ub \tag{II.D.4}
\]

\[
W_{Us,Us} = L^Us \tag{II.D.5}
\]

\[
W_{Us,Cb} = p_{Us,Cb} [C^s + C^b + \delta(\lambda \pi_s L^Us + \lambda(1 - \pi_s) L^Ub + (1 - \lambda) \pi_s L^Cs + (1 - \lambda)(1 - \pi_s) L^Cb)] + (1 - p_{Us,Cb}) [C^b - p_{\text{co}} + L^Us] \tag{II.D.6}
\]

\[
W_{Us,Cs} = L^Cs. \tag{II.D.7}
\]

**Cases 9 to 12:** In these cases, the first dealer is a connected buyer, i.e., \( k_1 j_1 =Cb \). The analysis is again almost identical to that in cases 1 to 4, except that if the first dealer is not matched with his successor, he is matched with a core dealer, which generates a surplus of \( (C^b - p_{\text{co}}) \). We obtain:

\[
W_{Cb,Ub} = C^b - p_{\text{co}} + L^Ub \tag{II.D.8}
\]

\[
W_{Cb,Us} = p_{Cb,Us} [C^s + C^b + \delta \lambda L^U + \delta (1 - \lambda) L^Cb] + (1 - p_{Cb,Us}) [C^b - p_{\text{co}} + L^Us] \tag{II.D.9}
\]

\[
W_{Cb,Cb} = C^b - p_{\text{co}} + L^Cb \tag{II.D.10}
\]

\[
W_{Cb,Cs} = p_{Cb,Cs} [C^s + C^b + \delta \lambda L^U + \delta (1 - \lambda) L^Cb] + (1 - p_{Cb,Cs}) [C^b - p_{\text{co}} + L^Cs] \tag{II.D.11}
\]

**Cases 13 to 16:** In these cases, the first dealer is a connected seller, i.e., \( k_1 j_1 = Cs \). The analysis is again almost identical to that in cases 9 to 12, except that if the first dealer is not matched with his successor, he is matched with a core dealer, which generates a surplus of
\((C^s + p^{cos}).\)

\[
W_{C^s,Ub} = p^{C^s,Ub} [C^s + C^b + \delta(\lambda\pi_s L^{Us} + \lambda(1 - \pi_s)L^{Ub} + (1 - \lambda)\pi_s L^{Cs} + (1 - \lambda)(1 - \pi_s)L^{Cb})] + (1 - p^{C^s,Ub})[C^s + p^{cos} + L^{Ub}]
\] (II.D.12)

\[
W_{C^s,Us} = p^{cos} + C^s + L^{Us}
\] (II.D.13)

\[
W_{C^s,Cb} = p^{C^s,Cb} [C^s + C^b + \delta(\lambda\pi_s L^{Us} + \lambda(1 - \pi_s)L^{Ub} + (1 - \lambda)\pi_s L^{Cs} + (1 - \lambda)(1 - \pi_s)L^{Cb})] + (1 - p^{C^s,Cb})[C^s + p^{cos} + L^{Cb}]
\] (II.D.14)

\[
W_{C^s,Cs} = p^{cos} + C^s + L^{Cs}.
\] (II.D.15)

Now, consider \(L^{kj}(P_m, \delta)\), namely the discounted future gains from trade when a dealer with type type \(kj\) is not matched with his predecessor. We have:

\[
L^{Ub} = \delta[\lambda\pi_s W^{Ub,Us} + \lambda(1 - \pi_s)W^{Ub,Ub} + (1 - \lambda)\pi_s W^{Ub,Cs} + (1 - \lambda)(1 - \pi_s)W^{Ub,Cb}]
\] (II.D.16)

\[
L^{Us} = \delta[\lambda W^{Us,Ub} + (1 - \lambda)W^{Us,Cb}]
\] (II.D.17)

\[
L^{Cb} = \delta[\lambda\pi_s W^{Cb,Us} + \lambda(1 - \pi_s)W^{Cb,Ub} + (1 - \lambda)\pi_s W^{Cb,Cs} + (1 - \lambda)(1 - \pi_s)W^{Cb,Cb}]
\] (II.D.18)

\[
L^{Cs} = \delta[\lambda W^{Cs,Ub} + (1 - \lambda)W^{Cs,Cb}].
\] (II.D.19)

Equations (II.D.1) to (II.D.19) define a linear system of 20 equations and 20 unknowns. Despite its size, this system has a particularly simple structure, since all the \(W^{k1j1,k2j2} \)
depend only on the $L^{kj}$, and conversely. To solve for this system, define:

\[
X = \begin{pmatrix}
W^{Ub, Ub} \\
W^{Ub, Us} \\
W^{Ub, Cb} \\
W^{Ub, Cs} \\
W^{Us, Ub} \\
W^{Us, Us} \\
W^{Us, Cb} \\
W^{Us, Cs} \\
W^{Cb, Ub} \\
W^{Cb, Us} \\
W^{Cb, Cb} \\
W^{Cb, Cs} \\
W^{Cs, Ub} \\
W^{Cs, Us} \\
W^{Cs, Cb} \\
W^{Cs, Cs} \\
L^{Ub} \\
L^{Us} \\
L^{Cb} \\
L^{Cs}
\end{pmatrix}
\]

\[
B = \begin{pmatrix}
0 \\
p^{Ub, Us}(C^s + C^b) \\
0 \\
p^{Ub, Cs}(C^s + C^b) \\
p^{Us, Ub}(C^s + C^b) \\
0 \\
p^{Us, Cs}(C^s + C^b) \\
0 \\
0 \\
p^{Cb, Us}(C^b) + (1 - p^{Cb, Us})(C^b - p^{cos}) \\
C^b - p^{cos} \\
p^{Cb, Cs}(C^b) + (1 - p^{Cb, Cs})(C^b - p^{cos}) \\
p^{Cs, Ub}(C^s + C^b) + (1 - p^{Cs, Ub})(C^s + p^{cos}) \\
C^s + p^{cos} \\
p^{Cs, Cs}(C^s + C^b) + (1 - p^{Cs, Cs})(C^s + p^{cos}) \\
C^s + p^{cos} \\
0 \\
0 \\
0 \\
0
\end{pmatrix}
\]
Finally, define $O_{16}$ a 16 $\times$ 16 matrix with only zeroes, and $O_4$ a 4 $\times$ 4 matrix with only zeroes.

Then we can write:

\[
A = \begin{pmatrix} O_{16} & A_1 \\ A_2 & O_4 \end{pmatrix}
\]

Now, our system of 20 equations and 20 unknowns can be written as:

\[
A.X + B = X,
\]

and we can solve for $X$ by inverting this equation:

\[
X = (Id - A)^{-1}.B
\]
Thus, inverting $I - A$, we can express $X$ as a function of the probabilities defining a specific matching plan. The resulting expression is very long, but relatively simple once we compute $W(P_m)$. In this way, we can easily derive the expression for the expected gains from trade per period associated with a particular equilibrium regime when $\delta \to 1$. We denote by $W(\Sigma)$ the expected gains from trade per period when $\delta$ goes to one for the matching plan corresponding to the strategy profile $\Sigma$.

**Step 3: Proof that the ACD regime is inefficient.** The matching plan corresponding to the strategy profile in the ACD regime is $p_{Ub,Us} = p_{Ub,Cs} = p_{Us,Ub} = p_{Us,Cb} = p_{Cb,Us} = p_{Cs,Ub} = 1$ and $p_{Cb,Cs} = p_{Cs,Cb} = 0$. Using (II.D.21), we solve $W(k_1j_1,k_2j_2)$ and $L^{kj}$ in this case for all possible values of $(k_1j_1, k_2j_2)$ and $kj$ (we omit the analytical formulas for brevity). We can then compute $W(\Sigma^{ACD})$ by taking the limit of $(1 - \delta)W(k_1j_1,k_2j_2)$ when $\delta \to 1$ for an arbitrarily chosen value of $(k_1j_1, k_2j_2)$. After some tedious algebra, we obtain:

$$W(\Sigma^{ACD}) = \left( Cb - p^{cos} \right)(1 - \lambda(1 - \lambda\pi_s)) + (Cs + p^{cos})(1 - \lambda\pi_s(1 - \lambda))\pi_s - (Cs + Cb)\lambda\pi_s^2(1 - \lambda(1 - \lambda)^2\pi_s) \over (1 + \pi_s)(1 - \lambda\pi_s(2 - \lambda - \lambda\pi_s(1 - \lambda)^2))$$

(II.D.22)

To show that the ACD regime is inefficient, consider equation (II.D.9), which gives the expected gains from trade when $(k_1j_1, k_2j_2) = (Cb, Us)$. In the ACD regime, we have $p_{Cb,Us} = 1$ because the connected buyer makes an offer that is accepted by the unconnected seller. To show that this is inefficient, let us differentiate $W^{Cb,Us}$ with respect to $p_{Cb,Us}$:

$$\frac{\partial W^{Cb,Us}}{\partial p_{Cb,Us}} = \frac{g_{Cb,Us}(\delta)}{1 - \delta}.$$  

(II.D.23)

where

$$g_{Cb,Us}(\delta) = (Cs + p^{cos})(1 - \delta) + \lambda\delta\hat{L}^{Ub} + (1 - \lambda)\delta\hat{L}^{Cb} - \hat{L}^{Us},$$

where we define $\hat{L}^{kj}(\delta) = (1 - \delta)L^{kj}(\delta)$

If $g_{Cb,Us}(\delta) < 0$, then the social planner can increase the expected gains from trade when $(k_1j_1, k_2j_2) = (Cb, Us)$ relative to the expected gains from trade in the ACD regime by

\footnote{Note that the aggregate gains from trade among peripheral dealers must be equal to the sum of the total trading surplus obtained by each type of peripheral dealers, as derived in (102) in the text. Thus, the expression below gives us the expression for $W(\Sigma)$ in (102) in the text.}
lowering the probability that the connected buyer trades with the unconnected seller. Thus, we show that $g_{Cb,Us}(\delta) < 0$.

We have:

$$g'_{Cb,Us}(\delta) = -(C^s + p^{cos}) + \lambda \hat{L}^{Ub} + (1 - \lambda) \hat{L}^{Cb} + \delta \left[ \lambda \frac{\partial \hat{L}^{Ub}}{\partial \delta} + (1 - \lambda) \frac{\partial \hat{L}^{Cb}}{\partial \delta} - \frac{\partial \hat{L}^{Us}}{\partial \delta} \right].$$  \hspace{1cm} (II.D.24)

Using the (omitted) analytical expressions of $\hat{L}^{Ub}$, $\hat{L}^{Cb}$, and $\hat{L}^{Us}$, and considering $\delta \to 1$, we obtain:

$$g'_{Cb,Us}(1) = \lambda (C^b - p^{cos})(1 - \pi_s) \frac{1}{1 - \lambda \pi_s (2 - \lambda - (1 - \lambda)^2 \lambda \pi_s)} > 0.$$ \hspace{1cm} (II.D.25)

Moreover, $g_{Cb,Us}(1) = 0$ because all the values of $\hat{L}^{jk}$ converge to $W(\Sigma^{ACD})$ when $\delta \to 1$. This remark and the fact that $g'_{Cb,Us}(1) > 0$ imply that $g(\delta) < 0$ for $\delta$ below 1 and sufficiently close to 1.

**Step 4: Proof that the ICB regime is inefficient.** The matching plan corresponding to the strategy profile in the ICB regime is: $p^{Ub,Us} = p^{Ub,Cs} = p^{Us,Ub} = p^{Cs,Ub} = 1$ and $p^{Us,Cb} = p^{Cb,Us} = p^{Cb,Cs} = p^{Cs,Cb} = 0$. Following the same method as in Step 2, we obtain:

$$W(\Sigma^{ICB}) = \frac{(C^b - p^{cos})(1 - \lambda (1 - \pi_s)) + (C^s + p^{cos})(1 - \lambda (1 - \lambda)^2 \pi_s) - (C^s + C^b) \lambda \pi_s^2}{(1 + \pi_s)(1 - \lambda \pi_s)}.$$ \hspace{1cm} (II.D.26)

To show that the ICB regime is inefficient, we apply the same method as that used in Step 2 for the ACD regime. In this case, we show that the central planner can improve gains from trade in state $(k_1j_1, k_2j_2) = (Us,Cb)$ relative to the gains from trade obtained in this state in the ICB regime. Using the expression for $W^{Us,Cb}$ obtained in Step 1, we deduce that:

$$\frac{\partial W^{Us,Cb}}{\partial p^{Us,Cb}} = \frac{g_{Us,Cb}(\delta)}{1 - \delta}.$$ \hspace{1cm} (II.D.27)
where
\[ g_{Us,Cb}(\delta) = (1-\delta)(C^s+C^b)+\delta[\lambda\pi_s\hat{L}^Us+\lambda(1-\pi_s)\hat{L}^Ub+(1-\lambda)\pi_s\hat{L}^Cs+(1-\lambda)(1-\pi_s)\hat{L}^Cb] - \hat{L}^Cb. \]  
(II.D.28)

Using the (omitted) analytical expressions of \( \hat{L}^Us, \hat{L}^Ub, \hat{L}^Cs, \) and \( \hat{L}^Cb, \) we obtain that:
\[ g_{Us,Cb}(\delta) = (1-\delta)[C^s+p^{cos}+(1-\delta)(C^b-p^{cos})] > 0. \]  
(II.D.29)

Thus, \( \frac{\partial W_{Us,Cb}}{\partial p_{Us,Cb}} > 0. \) This means that, from an efficiency standpoint, unconnected sellers make offers that have a too small chance of being accepted by connected buyers in an ICB equilibrium, since gains from trade can be increased by raising this likelihood. Thus, the ICB regime is inefficient.

**Step 5: Proof that the \( FB^- \) regime is efficient.** The matching plan corresponding to the strategy profile \( \Sigma^{FB^-} \) is: \( p^{Ub,Us} = p^{Ub,Cs} = p^{Us,Cb} = p^{Cs,Ub} = 1 \) and \( p^{Cb,Us} = p^{Cb,Cs} = p^{Cs,Cb} = 0. \) Using the same method as in the previous section, we obtain:
\[ W(\Sigma^{FB^-}) = \frac{(C^b-p^{cos})(1-\lambda+\lambda\pi_s(1-\pi_s))+(C^s+p^{cos})\pi_s(1-\lambda\pi_s)}{(1+\pi_s)(1-\lambda\pi_s)}. \]  
(II.D.30)

To show that this regime is efficient, we consider the derivative of each \( W^{k_1j_1,k_2j_2} \) with respect to \( p^{k_1j_1,k_2j_2}. \) As in Steps 2 and 3, we define \( g_{k_1j_1,k_2j_2}(\delta) \) as:
\[ g_{k_1j_1,k_2j_2}(\delta) = (1-\delta)\frac{\partial W^{k_1j_1,k_2j_2}}{\partial p^{k_1j_1,k_2j_2}}. \]

Thus, the sign of \( g_{k_1j_1,k_2j_2}(\delta) \) is the sign of \( \frac{\partial W^{k_1j_1,k_2j_2}}{\partial p^{k_1j_1,k_2j_2}} \) at the considered matching plan (here the one corresponding to the strategy profile \( FB^- \)). Proceeding as in Steps 2 and 3, we have
\[ g_{k_1 j_1, k_2 j_2}(1) = 0 \] and:

\[
\begin{align*}
g_{U_b, U_s}(1) &= g'_{U_b, C_s}(1) = -\frac{(1 - \lambda)(C^b - p^{co})}{1 - \lambda \pi_s} < 0 \quad \text{(II.D.31)} \\
g_{U_s, U_b}(1) &= -\frac{(C^b - p^{co})(1 - \pi_s) + (C^s + p^{co})(1 - \lambda \pi_s)}{1 - \lambda \pi_s} < 0 \quad \text{(II.D.32)} \\
g_{U_s, C_b}(1) &= -(C^s + p^{co}) < 0 \quad \text{(II.D.33)} \\
g'_{C_b, U_s}(1) &= g'_{C_b, C_s}(1) = \frac{\lambda(C^b - p^{co})(1 - \pi_s)}{1 - \lambda \pi_s} > 0 \quad \text{(II.D.34)} \\
g_{C_s, U_b}(1) &= -\frac{(C^b - p^{co})(1 - \pi_s)}{1 - \lambda \pi_s} < 0 \quad \text{(II.D.35)} \\
g_{C_s, C_b}(1) &= 0. \quad \text{(II.D.36)}
\]

Now consider, for instance, the case in which \((k_1 j_1, k_2 j_2) = (C_b, U_s)\). The allocation prescribed by \(FB^-\) implies that the connected buyer should not be matched with the unconnected seller \((p^{C_b, U_s} = 0 \text{ with } FB^-)\). As \(g'_{C_b, U_s}(1) > 0\) and \(g_{C_b, U_s}(1) = 0\), we deduce that \(g_{C_b, U_s}(\delta) < 0\) for \(\delta\) below and sufficiently close to 1. Thus, \(\frac{\partial W_{C_b, U_s}}{\partial p_{C_b, U_s}} < 0\) for all \(\delta\) below and infinitesimally close to 1. This means that setting \(p^{C_b, U_s} = 0\) is indeed socially optimal.

Proceeding in the same way in all other cases, we reach the conclusion that the matching plan corresponding to \(\Sigma^{FB^-}\) is efficient.

### E Proof that \(W(\Sigma^*)\) decreases in \(\lambda\)

This section shows that the welfare function \(W(\Sigma^*)\) decreases in \(\lambda\). The expressions for welfare are given in II. D for \(\Sigma^* \in \{\Sigma^{ACD}, \Sigma^{ICB}, \Sigma^{ICS}\}\). For instance, \(W(\Sigma^{ICB})\) is given in (II.D.26). We just consider the case in which \(\alpha^{pe} < 1/2\) because the analysis for other cases is identical. In this case, we have:

\[
W(\Sigma^{ACD}) = \frac{(C^b - p^{co})(1 - \lambda(1 - \lambda \pi_s)) + (C^s + p^{co})(1 - \lambda \pi_s(1 - \lambda)) \pi_s - (C^s + C^b) \lambda \pi_s^2(1 - \lambda(1 - \lambda)^2 \pi_s)}{(1 + \pi_s)(1 - \lambda \pi_s(2 - \lambda - \lambda \pi_s(1 - \lambda)^2))}.
\]

Thus:
\[
\frac{\partial W(\Sigma^{ACD})}{\partial \lambda} = -(C^b - p^{cos})(1 - \pi_s)(1 - \pi_s(1 - \lambda(2\pi_s - \lambda(1 + \pi_s(6 - 8\lambda + 3\lambda^2 + (1 - \lambda)^2(1 - 2\lambda)\pi_s)))}) \leq 0.
\]

Hence, \(W(\Sigma^{ACD})\) decreases in \(\lambda\).

Welfare in the ICB equilibrium is given by:

\[
W(\Sigma^{ICB}) = (C^b - p^{cos})(1 - \pi_s) + (C^s + p^{cos})(1 - \lambda(1 - \lambda)^2)\pi_s - (C^s + C^b)\lambda\pi_s^2.
\]

Thus:

\[
\frac{\partial W(\Sigma^{ICB})}{\partial \lambda} = p^{cos}(1 - \pi_s(3 - \pi_s - \lambda(2 - \lambda\pi_s))) - C^b(1 - \pi_s)^2 - C^s(1 - \lambda(2 - \lambda\pi_s))
\]

After some rearrangements, we can rewrite this expression as:

\[
\frac{\partial W(\Sigma^{ICB})}{\partial \lambda} = \frac{x + y}{(1 + \pi_s)(1 - \lambda\pi_s)^2} \left[ p^{cos} - \left( \omega'^{C^b} - (1 - \omega')C^s \right) \right]
\]

with \(x = (1 - \pi_s)^2\)
\(y = -\pi_s(1 - \lambda)(1 - \lambda(3 - 2\lambda\pi_s))\)
\(\omega' = \frac{x}{x + y}\).

Remember that in the ICB equilibrium it must be the case that \(p^{cos} \leq \omega_s C^b - (1 - \omega_s)C^s\).

We have:

\[
\omega' - \omega_s = \frac{(1 - \pi_s)(1 - \lambda)(1 - \pi_s(1 - \lambda(\pi_s - \lambda(3 - 2\lambda\pi_s))))}{(1 - \lambda\pi_s(2 - \lambda))(x + y)}.
\]

There are two cases to consider.

**Case 1.** \(x + y > 0\). In this case, we have \(\omega' > \omega_s\). Thus, as \(p^{cos} \leq \omega_s C^b - (1 - \omega_s)C^s\) in an ICB equilibrium, we have \(p^{cos} \leq \omega'^{C^b} - (1 - \omega')C^s\). This is a sufficient (and necessary) condition for \(\frac{\partial W(\Sigma^{ICB})}{\partial \lambda} \leq 0\) when \(x + y > 0\).
Case 2. $x + y < 0$. In this case, $\frac{\partial W(\Sigma_{ICB})}{\partial \lambda} \leq 0$ if and only if $p^{co} \leq \omega C^b - (1 - \omega')C^s$. This inequality is always satisfied because $p^{co} > -C^s$ and $\omega' < 0$ if $x + y < 0$.

In sum, we have shown that in the ACD and the ICB regimes, agents’ welfare decreases with $\lambda$. In the ACD regime, $p^{co} \geq \omega_s C^b - (1 - \omega_s)C^s$ while in the ICB $p^{co} < \omega_s C^b - (1 - \omega_s)C^s$. The switch from ACD to ICB takes place when $\lambda$ is large enough because $\omega_s$ increases in $\lambda$. The switch takes place when $\lambda$ is strictly higher than the threshold value for which $p^{co} = \omega_s C^b - (1 - \omega_s)C^s$. At this point, a small increase in $\lambda$ triggers a discontinuous jump in $W(\Sigma)$ from $W(\Sigma_{ACD})$ to $W(\Sigma_{ICB})$. This jump is negative because when $p^{co} = \omega_s C^b - (1 - \omega_s)C^s$, we obtain:

$$W(\Sigma_{ICB}) - W(\Sigma_{ACD}) = -\frac{(C^b + C^s)(1 - \lambda)^2\lambda^3(1 - \pi_s)\pi_s^2(1 - \lambda\pi_s(1 + \lambda(1 - \lambda)))}{(1 + \pi_s)(1 - \lambda\pi_s)(1 - \lambda\pi_s(2 - \lambda))(1 - \lambda\pi_s(2 - \lambda - \lambda\pi_s(1 - \lambda)^2))} < 0.$$ 

We deduce that $W(\Sigma^*)$ decreases in $\lambda$ everywhere for $\alpha^{pe} \leq \frac{1}{2}$. A similar reasoning shows that this is also the case when $\alpha^{pe} > \frac{1}{2}$.

F Details on numerical simulations

In this section we give more details on the procedure followed to generate the simulated data used in Section 5.2 of our paper.

We simulate 250 “trading days”. Each day is characterized by (i) a specific realization of $(\alpha^{pe}, \alpha^{co})$ and (ii) the number of peripheral dealers $N$ “arriving” on this day (i.e., chosen to trade on this day). Specifically, on each day, a new $\alpha^{pe}$ (or $\alpha^{co}$) is drawn from a Normal distribution with mean 0.5 and standard deviation 0.2, truncated to the interval $[0, 1]$. Moreover, $N$ is equal to $1 + \tilde{X}$, where $\tilde{X}$ follows a Geometric distribution with average 100. This is equivalent to assuming that peripheral dealers in our model can trade (as described in the model) until a random stopping time. All these variables are drawn independently from each other in a given day and across days.

The other parameters of the model are constant over all days and set as follows: $C_b = C_s = 1; \lambda = 0.4; L = 1.5; \beta = 0.5$. Moreover, we assume that the distribution $\Phi$ of core dealers’ inventory shock, $\epsilon_{i3}$, is a Gaussian distribution with mean 0 and standard deviation 0.8. Last, we focus on the thick core case: $\kappa \to +\infty$. 38
In each day \( d \in \{1, \ldots, 250\} \), we first determine which equilibrium regime, \( \Sigma^*_d \), obtains given the realization of \((\alpha^{pe}, \alpha^{co})\) for this day. We then assume that the \( N \) selected peripheral dealers for this day are lined up, as described in the model, with types drawn according to probabilities \( \pi_b(\alpha^{pe}) \) and \( \pi_s(\alpha^{pe}) \) (the first dealer is a seller with probability \( \alpha^{pe} \) and is unconnected with probability \( \lambda \)). We label these dealers: dealer 1, 2, ..., \( N \). Starting from dealer 1, we then implement the following algorithm to generate a sequence of trades and prices:

1) We record the transaction between dealer 1 and his client. For instance, if dealer 1 is a connected buyer, we record that he sold one unit at price \( \text{ask}^C \) to a client.

2)a) If the type of dealer 2 is such that, in the equilibrium regime \( \Sigma^*_d \), a trade occurs between dealers 1 and 2, we record two transactions: (i) a transaction between dealer 2 and his client and (ii) a transaction between dealers 1 and 2. These transactions are those obtained in the equilibrium regime \( \Sigma^*_d \) and take place at the equilibrium prices obtained in this regime. For instance, if dealer 1 is a connected buyer and dealer 2 is an unconnected seller, they trade together if \( \Sigma^*_d = \Sigma^{ACD} \) or \( \Sigma^*_d = \Sigma^{ICS} \). In this case, we record two transactions: a transaction at price \( \text{bid}^U \) between the unconnected seller and a client, and a transaction at price \( p^{sc}_C \) between the connected seller and the unconnected buyer. We then restart at step 1 with dealer 3 playing the role of dealer 1.

2)b) If (i) the type of dealer 2 is such that, in the equilibrium regime \( \Sigma^*_d \), no trade occurs between dealers 1 and 2 and (ii) dealer 1 is connected then we record a trade at price \( p^{co*} \) between dealer 1 and a core dealer and between the core dealer and a client. Indeed, in all cases in our model, a connected dealer trades with a core dealer if his offer to a peripheral dealer is rejected. We then restart at step 1 with dealer 3 playing the role of dealer 1.

2)c) If (i) the type of dealer 2 is such that, in the equilibrium regime \( \Sigma^*_d \), no trade occurs between dealers 1 and 2 and (ii) dealer 1 is not connected then we do not record a new trade. We then restart at step 1 with dealer 2 playing the role of dealer 1.

The algorithm stops when dealer \( N \) (i) accepts an offer from dealer \( N - 1 \), (ii) rejects the offer from dealer \( N - 1 \) or (iii) receives no offer (the only three possibilities). Finally, we include 5 trades between core dealers at price \( p^{co*} \) so that there are some core-core trades in the data. The actual number of such trades does not matter for our conclusions in Section
5.2. It just guarantees that there are observations in which the dummy variables used in our regressions in Section 5.2 are all zero (this corresponds to transactions between core dealers).

Proceeding in this way, we obtain a set of transactions and prices both in the D2D and the D2C markets (in fact for each trade between dealers, there are two corresponding trades in the D2C market, as in our model). Trades are ordered sequentially and, for each trade, we record (i) the price of the trade; (ii) the type of the seller (client, unconnected dealer, connected dealer, core dealer); and (iii) the type of the buyer. In addition, we record peripheral dealers’ aggregate inventory positions on the day on which trades happen ($z_{0e}^{pe}$ and $z_{0e}^{co}$). This generates the dataset of simulated trades and prices that we use to estimate (32) and (33) in Section 5.2. As an illustration, Figure II.F.1 below gives a snapshot of the first lines of this dataset.

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<th>buyer_id</th>
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<th>buyer_type</th>
<th>price</th>
<th>day_epe</th>
<th>day_eco</th>
<th>day_vco</th>
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<td>0.676</td>
<td>0.946</td>
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</tr>
<tr>
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<td>2</td>
<td>3</td>
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<td>Core</td>
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<td>0.676</td>
<td>0.946</td>
<td>-0.61</td>
</tr>
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<td>0.676</td>
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</tr>
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<td>1</td>
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<td>0.676</td>
<td>0.946</td>
<td>-0.61</td>
</tr>
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Figure II.F.1: First twenty lines of the simulated dataset.
G Proofs of Implications 1 to 5

G.1 Proof of Implication 1

We first prove that \( \bar{p}_{C,U} \geq \bar{p}_{U,U} \geq \bar{p}_{U,C} \). \( \bar{p}_{C,U} \) is a weighted average of \( p_C \) and \( p_U \), while \( \bar{p}_{U,U} \) is a weighted average of \( p_U \) and \( p_U \). Using Proposition 2 (or Figure 5), we have \( p_C \geq p_U \). Hence, \( \bar{p}_{C,U} \geq \bar{p}_{U,U} \). A symmetric reasoning shows that \( \bar{p}_{U,U} \geq \bar{p}_{U,C} \).

For the second part of Implication 1, we just consider the case \( \alpha_{pe} > 1/2 \) because the other cases are symmetric. Thus, we must show that \( \bar{p}_{C,U} - \bar{p}_{U,U} \) increases in \( \alpha_{pe} \) when \( \alpha_{pe} > 1/2 \).

Using the stationary probabilities derived in the Online Appendix II. B, in each period an unconnected buyer accepts an offer from an unconnected seller at price \( p_U \) with probability \( \left( \mu_3 \pi b \right) \), and an unconnected seller accepts an offer from an unconnected buyer at price \( p_U \) with probability \( \left( \mu_7 \lambda \right) \). We deduce that:

\[
\bar{p}_{U,U} = \frac{\mu_3 \times \pi b \lambda}{\mu_3 \times \pi b \lambda + \mu_7 \times \lambda} p_U + \frac{\mu_7 \times \lambda}{\mu_3 \times \pi b \lambda + \mu_7 \times \lambda} p_U. \tag{II.G.1}
\]

A similar reasoning yields:

\[
\bar{p}_{C,U} = \frac{\mu_1 \times \pi b \lambda}{\mu_1 \times \pi b \lambda + \mu_7 \times (1 - \lambda)} p_C + \frac{\mu_7 \times (1 - \lambda)}{\mu_1 \times \pi b \lambda + \mu_7 \times (1 - \lambda)} p_U = p^{co}. \tag{II.G.2}
\]

Using these expressions, after some algebra, we obtain:

\[
\bar{p}_{C,U} - \bar{p}_{U,U} = \frac{\lambda(1 - \pi b)}{1 - \lambda} \times \frac{1 - \pi b(1 - \lambda \pi b(1 - \lambda)^2)}{1 - \pi b(1 - \lambda(1 + \pi b)(1 - \lambda)^2)} \tag{II.G.3}
\]

The first term in the product is decreasing in \( \pi b \). The derivative of the second term with respect to \( \pi b \) is:

\[
\frac{-\lambda(1 - \lambda)^2(1 - \lambda \pi b^2(1 - \lambda)^2)}{[1 - \pi b(1 - \lambda(1 + \pi b)(1 - \lambda)^2)]^2} < 0. \tag{II.G.4}
\]

Thus, \( \bar{p}_{C,U} - \bar{p}_{U,U} \) decreases with \( \pi b \), and hence increases in \( \alpha_{pe} \).

G.2 Proof of Implication 2

The results follow straightforwardly from (31).
G.3 Proof of Implication 3

We consider the case $\alpha^{pe} \geq 1/2$, the other case being symmetric. We define $I_s$ (resp. $I_b$) the probability that an unconnected seller (resp. buyer) ends with an inventory.

Consider the ACD Equilibrium first. Since $\pi_s = 1$, unconnected buyers find a counterparty with probability 1 and do not hold inventories after trading, hence $I_b = 0$. The probability in any given period that an unconnected seller makes an offer and this offer is rejected is $\mu_3(1 - \pi_b)$. Since the probability of having an unconnected seller arriving in a given period is $\lambda \alpha^{pe} = \lambda/(1 + \pi_b)$, we have:

$$I_s = \frac{\mu_3(1 - \pi_b)^{1 + \pi_b}}{\lambda_{AB}} = \frac{(1 - \pi_b)(1 - \pi_b\lambda(1 - \lambda)^2)}{1 - \pi_b\lambda(2 - \lambda(1 + \pi_b(1 - \lambda)^2))}. \quad (II.G.5)$$

We then show that this analytical expression is decreasing in $\pi_b$ (hence increasing in $z^{pe}_0$) and increasing in $\lambda$.

We can write $I_s$ as the product of $f_{Is}$ and $g_{Is}$, with:

$$f_{Is} = 1 - \pi_b(1 - \pi_b\lambda(1 - \lambda)^2), \quad g_{Is} = \frac{1 - \pi_b}{1 - \pi_b\lambda(2 - \lambda(1 + \pi_b(1 - \lambda)^2))}. \quad (II.G.6)$$

The derivative of $f_{Is}$ with respect to $\pi_b$ is equal to $-1 + 2\lambda\pi_b(1 - \lambda)^2$, which is negative. The derivative of $g_{Is}$ with respect to $\pi_b$ has the sign of $-(1 - \lambda)^2[1 + (2 - \pi_b)\pi_b\lambda^2]$, which is also negative. This shows that $I_s$ decreases in $\pi_b$, and hence increases in $z^{pe}_0$.

To show that $I_s$ increases in $\lambda$, we differentiate with respect to $\lambda$ and obtain:

$$\frac{\partial I_s}{\partial \lambda} = \frac{\pi_b\lambda(1 - \pi_b)h_{Is}(\pi_b)}{[1 - \pi_b\lambda(2 - \lambda(1 + \pi_b(1 - \lambda)^2))]^{2}} \quad (II.G.7)$$

with $h_{Is}(\pi_b) = 2 - \pi_b - \lambda\pi_b[5 - 4\lambda + \pi_b^2\lambda(1 - \lambda)^3 - \pi_b(2 - \lambda - \lambda^2)] \quad (II.G.8)$

We need to show that $h_{Is}(\pi_b)$ is always positive. We first compute that $h''_{Is}(\pi_b) = 2\lambda[2 - \lambda(1 + 3\pi_b(1 - \lambda)^3 + \lambda)]$. It is easy to show that this term is positive using that $\pi_b \leq 1$ and $\lambda \leq 1/2$. This implies that for any $\pi_b$ we have $h'_{Is}(\pi_b) \leq h'_{Is}(1)$. We compute that $h'_{Is}(1) = -1 - \lambda - \lambda^2(1 - \lambda)[1 - 3\lambda(2 - \lambda)]$. It is easily shown that this quantity is negative for any $\lambda \in [0, 1]$. This implies that for any $\pi_b$ we have $h_{Is}(\pi_b) \geq h_{Is}(1)$. Finally, we compute
that \( h_{Is}(1) = (1 + \lambda)(1 - \lambda)^4 > 0 \), from which we deduce that \( h_{Is}(\pi_b) \geq 0 \) for any \( \pi_b \). This concludes the proof that \( I_s \) increases in \( \lambda \).

Consider now the ICS Equilibrium. Using the same reasoning as above, we obtain:

\[
I_s = \mu_3(1 - \pi_b) \frac{1 + \pi_b}{\lambda} = \frac{(1 - \pi_b)^2}{1 - \pi_b \lambda} \quad \text{(II.G.9)}
\]

\[
I_b = \mu_7(1 - \lambda) \frac{1 + \pi_b}{\pi_b \lambda} = \frac{(1 - \lambda)^2}{1 - \pi_b \lambda} \quad \text{(II.G.10)}
\]

Simple derivations then shows that \( I_s \) decreases in \( \pi_b \) and increases in \( \lambda \), while \( I_b \) increases in \( \pi_b \) and decreases in \( \lambda \), which concludes the proof.

### G.4 Proof of Implication 4

We need to show that bid and ask prices in the D2C markets are ranked as follows: (i) If \( z_0^{pe} > 0 \), then \( bid^C = bid^{co} > bid^U \) and \( ask^C < ask^U < ask^{co} \); (ii) If \( z_0^{pe} < 0 \), then \( ask^C = ask^{co} < ask^U \) and \( bid^C > bid^U > bid^{co} \).

We focus on the case \( \alpha^{pe} > 1/2 \), the other case being symmetric. Using (29) and (30), we need to show that \( P_s^{co} = P_s^C > P_s^U \) and \( P_b^C < P_b^U < P_b^{co} \). These inequalities can be proved by using the analytical expressions (74) to (77), but it is easier to use the following economic argument:

When \( \alpha^{pe} > 1/2 \) the connected sellers only trade at price \( p^{cos} \), either with unconnected dealers or with core dealers. This shows that \( P_s^{co} = P_s^C \). Unconnected sellers are as likely as connected sellers to receive offers at price \( P_b^U = p^{cos} \), and otherwise make offers at price \( P_s^U < p^{cos} \) or keep an inventory at cost \(-C^s < p^{cos} \). Hence, they necessarily have \( P_s^U < P_s^{co} \). By the same reasoning, we necessarily have \( P_b^C < P_b^U \). Finally, unconnected buyers never keep an inventory, and trade either at \( P_b^U < p^{cos} \), \( P_b^C = p^{cos} \), or \( P_b^U = p^{cos} \). Hence, the average price \( P_b^U \) is indeed strictly below \( P_b^{co} \).

Now consider the ICS Equilibrium. Connected sellers only trade in the core so that \( P_s^{co} = P_s^C \). Unconnected sellers only receive offers at price \( P_b^U = p^{cos} \), so that \( P_s^U < p^{cos} \). Unconnected buyers could ensure an average price of \( p^{cos} \) by deviating and making offers at price \( p^{cos} \) that would be accepted with probability 1. In an ICS Equilibrium, it is necessarily the case that when they make an offer they obtain an average price below \( p^{cos} \). The offers at
price \( p_s^U \) they receive from unconnected sellers are also below \( P^{co} \), so that \( P_b^U < P_b^{co} \). Finally, connected buyers are necessarily strictly better off than unconnected buyers in equilibrium, so that \( P_b^C < P_b^U \).

### G.5 Proof of Implication 5

(i) First, we show that \( S^C < S^{co} < S^U \) and that \( S^{co} < (1 - \lambda)S^C + \lambda S^U \) (for \( \lambda > 0 \) and \( \alpha^{pe} \neq 0 \)). Using (29) and (30), we obtain:

\[
S^k = ask^k - bid^k = 2L(1 - \beta) + \beta(P^k_b - P^k_s). \tag{II.G.11}
\]

The inequality \( S^C < S^{co} \) follows directly from Implication 4. Proving that \( S^{co} < S^U \) is more involved, as unconnected dealers on the uncrowded side offer better prices to their clients than core dealers. Assume that \( \alpha^{pe} \geq 1/2 \) (the analysis for \( \alpha^{pe} < 1/2 \) is symmetric). In this case, the equilibrium regime is either \( ACD \) or \( ICS \). If the equilibrium is \( ACD \), we obtain using (75) and (77) (and the equilibrium values of the stationary probabilities and \( \varphi_k \) appearing in these equations) that \( P^U_b - P^U_s = (C^s + p^{cos}) \times (a_1/b_1) \), with:

\[
a_1 = (1 - \pi_b)(1 - \lambda\pi_b)[1 - \lambda\pi_b](2 + \lambda - \pi_b(1 + \lambda)(1 - \lambda)^2 + \lambda\pi^2_b(1 - \lambda)^2)] \tag{II.G.12}
\]

\[
b_1 = (1 - \pi_b)(2 - 2\lambda - \lambda\pi_b(1 - \lambda)^2) \tag{II.G.13}
\]

Simple manipulations, omitted for brevity, show that \( a_1 \) and \( b_1 \) are both positive, so that indeed \( P^U_b - P^U_s > 0 \). Since \( P^{co}_b - P^{co}_s = 0 \), this shows that \( S^U > S^{co} \).

If the equilibrium is \( ICS \), (75) and (77) yield:

\[
P^U_b - P^U_s = \frac{a_2[C^s + p^{cos} + b_2[C^s + p^{cos}]]}{(1 - \lambda\pi_b)^2} \tag{II.G.14}
\]

with

\[
a_2 = (1 - \lambda)(1 - \lambda(2 - \pi_b(1 + \lambda - \pi_b))) \tag{II.G.15}
\]

\[
b_2 = (1 - \pi_b)(1 - \lambda\pi_b)[3 + \pi^2_b - (2 + \lambda)\pi_b] \tag{II.G.16}
\]

Again, it is easily shown that \( a_2 \) and \( b_2 \) are both positive, so that \( P^U_b - P^U_s \geq 0 \).

The inequality \( \lambda S^U + (1 - \lambda)S^C > S^{co} \) can be proved analytically, but we rather use the
following economic reasoning. We can first rewrite the inequality as:

\[(1 - \lambda)[P_s^C - p^co] + (1 - \lambda)[p^co - P^C_b] + \lambda[P^U_s - p^co] + \lambda[p^co - P^C_b] < 0. \tag{II.G.17}\]

This equation means that peripheral dealers on average do not trade at more favorable prices than \(p^co\). This is necessarily true. Indeed, the only reason why connected dealers and unconnected dealers on the uncrowded side trade at more favorable prices than \(p^co\) is that they exploit their market power over unconnected dealers on the crowded side of the market. Hence, the gains of dealers with market power are fully compensated by losses of dealers without market power. In addition, unconnected dealers end with an inventory with a positive probability, and the cost of inventories is less favorable than \(p^co\). Hence, in aggregate it has to be the case that peripheral dealers trade at prices less favorable than \(p^co\), so that (II.G.17) is satisfied and hence \(\lambda S^U + (1 - \lambda)S^C > S^co\).

(ii) The spread of core dealers is \(S^co = 2L(1 - \beta)\). Thus, it does not depend on \(z_0pe\) and \(z_{0co}\).

(iii) Finally, we show that \(S^C\) decreases with the size of peripheral dealers’ aggregate inventory, \(|z_{0pe}^p|\). We just consider the case \(\alpha^pe > 1/2\) (i.e., \(z_0^co\)) because the other case is symmetric. In the ACD Equilibrium we have \(S^C = 2L(1 - \beta) + \beta[P^C_b - p^co]\), and, using the expression for \(P^C_b\) (see (76)), we obtain after some algebra that:

\[S^C = 2L(1 - \beta) - \beta \frac{\lambda(1 - \pi_b)(1 - \lambda)(C^s + p^co)}{1 - \pi_b \lambda}. \tag{II.G.18}\]

As \(p^co\) decreases in \(z_0^co\) and \(\pi_b\) decreases in \(z_{0pe}^p\), we obtain that \(S^C\) decreases in \(z_{0pe}^p\) and increases in \(z_0^co\) when \(\alpha^pe > 1/2\).
Part III

Extensions and Robustness

This Appendix presents a number of extensions of the model and robustness checks. Section A allows unconnected and connected dealers to have any degree of connectedness to the core, not necessarily zero and infinity as in our main model. Section B shows that our assumptions on the sequential arrival of traders are isomorph to a particular search setup. Section C shows that the inventory shock to core dealers that we introduce in the main model plays the same role as assuming convex inventory costs for core dealers. Section D considers a variant in which bilateral transactions in the periphery are conducted via a double auction instead of take-it-or-leave-it offers. Section E relaxes the assumption that the fraction of unconnected dealers is lower than one half. Section F relaxes the assumption that dealers on the uncrowded side find a counterparty with probability 1.

A Heterogeneity of connections in the periphery

The main model features extreme heterogeneity in connectedness between the two types of dealers: connected dealers are perfectly connected to the core (they trade at the same price as core dealers), while unconnected dealers cannot trade with core dealers at all. In this section, we extend the model to allow for an arbitrary degree of heterogeneity: both “unconnected” and “connected” dealers are connected to core dealers but the latter have more connections than the former. We first describe how peripheral dealers trade with core dealers in Section A.1. Then, in Section A.2, we solve for the quotes posted by core dealers when they are contacted by connected or unconnected peripheral dealers. This analysis enables us to compute the expected price at which a peripheral dealer expects to trade with core dealers if he does not find a trading partner among peripheral dealers (see Section A.3). We then deduce, in Section A.4, that the nature of equilibria in the peripheral market is as in our baseline model and that, for this reason, our main implications are qualitatively unchanged.
A.1 A model of trading between core and peripheral dealers

We assume that connected (resp. unconnected) peripheral dealers have access to $n_C$ (resp. $n_U$) dealers in the core, with $n_U < n_C$. If a peripheral dealer decides to trade with core dealers, he contacts all the core dealers whom he has access to and requests quotes from them (at some point between dates 1 and 2). Each core dealer responds (is “active”) to this request with probability $p$ and does not (is “inactive”) with probability $q = 1 - p$. For instance, in reality, core dealers might choose not respond to a request when their inventory has already reached position limits. All core dealers respond simultaneously to a peripheral dealer’s request for a quote. They know the number of potential bidders (e.g., $n_C$ if the peripheral dealer is connected) but they do not know the actual number of active core dealers responding to a given request. If the peripheral dealer is a seller (buyer) and he receives at least one response, he trades with the dealer giving the highest (smallest) price, provided that this price is above (below) his inventory holding cost, $-C_s^* (C_b^*)$, if he does not trade. Otherwise, the peripheral dealer does not trade and incurs the inventory holding cost at date 3. Thus, in effect, we are modeling competition among core dealers as a first price auction in which the number of bidders is uncertain. We are referring to this auction as “The Request for Quote Game” (RFQ game).

The rest of the model is unchanged. In particular, after trading with peripheral dealers, core dealers can trade among each other at date 2 in a centralized market at the price $p^{co}$, determined as in our baseline model. Moreover, peripheral dealers trade bilaterally among each other at date 1 as described in the model.

A.2 Equilibrium of the Request for Quote’ Game

We now solve for the equilibrium of the RFQ game. Denote $\tilde{D}$ the number of active dealers, and $p_j = \Pr[\tilde{D} = j + 1 | \tilde{D} \geq 1]$. Note that $\tilde{D} \sim B(n + 1, p)$, where $n = n_U$ if core dealers are contacted by an unconnected peripheral dealers and $n = n_C$ if they are contacted by a connected peripheral dealer.

We just consider the case in which core dealers are contacted by a peripheral seller because

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13See Vogel (2019) and Glebkin, Shen, and Yueshen (2019) for models of requests for quotes in an OTC market.
the analysis for a peripheral buyer is symmetric. The payoff of a core dealer with the highest bid price, \(b\), is \(p^c - b\), provided that \(b \geq -C^s\). Indeed, a core dealer who acquires the asset from a peripheral dealer can then resell it in the core market at price \(p^c\). A core dealer whose offer is rejected obtains a payoff of zero. To solve for the equilibrium of the RFQ game in this case, we closely follow the analysis in Haviv and Milchtaich (2012), to which we refer the reader for more details. As is well known in this type of game, the equilibrium can only be in mixed strategies, with a mass of zero on each possible bid. Intuitively, the reason is that: (i) If any bid strictly smaller than \(p^c\) is played with a strictly positive probability then other dealers have an incentive to bid slightly below; (ii) Playing \(p^c\) with a strictly positive probability is not optimal because it brings zero profit, whereas bidding \(-C^s\) brings a profit of \(p^c + C^s\) with probability \(q^{n-1}\).

Hence, we consider a symmetric mixed strategy equilibrium in which core dealers respond to a seller’s request by picking a bid price, \(b\), randomly according to the cumulative distribution \(F(b) = \Pr[\tilde{b} \leq b] (f(b) = F'(b))\). We guess and verify that \(F\) is strictly increasing on an interval \([b, \tilde{b}]\) to be defined. It is also useful to define \(H = F^{-1}\), which is defined on \([0, 1]\). One way to think about \(H\) is the following: each dealer picks a virtual type \(x \rightarrow U[0, 1]\) and then plays \(H(x)\).

Under this approach, the probability, \(P(x)\), that a core dealer with virtual type \(x\) wins the RFQ game is

\[
P(x) = [q + px]^{n-1}. \tag{III.A.1}
\]

That is, the likelihood that a core dealer with virtual type \(x\) wins the RFQ game is the probability that all other dealers are either inactive or have a lower type. We deduce that the equilibrium expected payoff of a dealer with virtual type \(x\) is:

\[
P(x)[p^c - H(x)]. \tag{III.A.2}
\]

In a mixed strategy equilibrium, a core dealer has to be indifferent between each \(x\). That is, his expected payoff in equilibrium must be a constant, \(K\), for any \(x\). This is true in particular for \(x = 0\). Thus, \(K = q^{n-1}[p^c - H(0)]\). As this expected payoff is maximal for \(H(0) = -C^s\) (the monopoly price), we deduce that:
Conversely, if $x = 1$ then the dealer is sure to win the auction ($P(1) = 1$). His expected payoff is therefore $p^o - H(1)$. As this must also be equal to $K$, we deduce:

$$H(1) = p^o - K = p^o(1 - q^{n-1}) - q^{n-1}C^s.$$  

(III.A.4)

Thus, we have shown that $\bar{b} = -C^s$ and $\bar{b} = p^o(1 - q^{n-1}) - q^{n-1}C^s$. Finally, using the condition, $K = P(x)[p^o - H(x)]$, we can explicitly solve for $H(x)$. We obtain:

$$H(x) = p^o - \left(\frac{q}{q + px}\right)^{n-1} [p^o + C^s].$$ 

(III.A.5)

By definition, $F = H^{-1}$. Hence, we deduce from the previous equation that:

$$F(b) = H^{-1}(b) = \frac{q}{p} \left[\left(\frac{p^o + C^s}{p^o - b}\right)^{\frac{1}{n-1}} - 1\right].$$ 

(III.A.6)

It is easily checked that $F$ is strictly increasing in $b$, with $F(-C^s) = 0$ and $F(p^o(1 - q^{n-1}) - q^{n-1}C^s) = 1$. We have thus found a symmetric mixed strategy equilibrium of the RFQ game.

### A.3 Equilibrium payoffs

In order to embed the RFQ game into our model of bilateral trading in the peripheral market, we must compute the expected price at which a peripheral dealer expects to trade with core dealers if he does not find a trading partner in the peripheral market. Indeed, this expected price determines his outside option when he bargains with other peripheral dealers.

As in the previous section, we focus on the case of a peripheral seller for brevity (the case of a peripheral buyer is symmetric). To compute the expected price at which a peripheral dealer with a long position (a seller) unwinds his inventory if he does not trade with other peripheral dealers, we proceed as follows. First, we observe that the total expected payoff of core dealers who are contact by a peripheral seller must be equal to $n_k p K$ since (i) conditional on being active a core dealer’s expected profit is $K$, (ii) a core dealer is active with probability
\( p \), and the peripheral seller contact \( n_k \in \{n_C, n_U\} \) core dealers. In turn, the total expected payoff of core dealers must be equal to the difference between the resale price of the asset for core dealers, \( p^{co} \) time the probability that at least one core dealer is active and the expected payment (denoted \( \mathbb{E}[\text{Transfer}] \)) of the seller to core dealers (accounting for the fact that this payment is zero if no dealer is active). Thus:

\[
npK = [1 - q^n]p^{co} - \mathbb{E}[\text{Transfer}].
\]  

(III.A.7)

Using the expression for \( K \), derived in the previous section, we deduce that:

\[
\mathbb{E}[\text{Transfer}] = (1 - q^n)p^{co} - npK = p^{co}[1 - q^n - npq^{n-1}] - npq^{n-1}C^s.
\]  

(III.A.8)

We deduce that the expected price, \( p^{co}_{s,k} \), at which a seller with type \( k \in \{U, C\} \) unwinds his inventory if he does not trade in the peripheral market is:

\[
p^{co}_{s,k} = -q^nC^s + \mathbb{E}[\text{Transfer}] = -C^s + [1 - q^{nk} - n_kpq^{nk-1}][p^{co} + C^s].
\]  

(III.A.9)

This price is simply a weighted average of \( -C^s \) and \( p^{co} \). We obtain \( p^{co}_{s,k} \to p^{co} \) when \( n \to +\infty \) (perfect competition) or \( p \to 1 \) (Bertrand competition), and \( p^{co}_{s,k} \to -C^s \) when \( q \to 1 \) or \( n \in \{0,1\} \) (one or no connection). Our main model is equivalent to the extreme case in which \( n_C \to +\infty \) and \( n_U \in \{0,1\} \).

### A.4 Impact on the equilibrium in the periphery

With this more general model, when trading in the core a periphery dealer of type \( (j,k) \in \{B,S\} \times \{C,U\} \) obtains a price of \( p^{co}_{j,k} \). Most properties of the initial model still hold, in particular: (i) gains from trade are lower between connected dealers than between unconnected dealers; (ii) the outside option of unconnected dealers is always less valuable than the outside option of connected dealers; (iii) there is always a probability that a peripheral dealer ends with an inventory (if he doesn’t find anyone in the core).
The equilibrium of the peripheral market is solved exactly as in the main model (see Section 4.2 in the model). To give an example, let us consider the ACD equilibrium. Proceeding as in the proof of Lemma 2 and 3, we find that the following conditions must be satisfied in the ACD equilibrium:

\[
\begin{align*}
    p_C^s &= -V_U^b \\
    p_C^b &= V_s^U \\
    p_U^s &= -V_C^b \\
    p_U^b &= V_s^C \\
    V_s^C &= \lambda \pi b p_s^C + (1 - \lambda \pi b)p_{s,C}^{co} \\
    V_b^C &= -\lambda \pi s p_b^C - (1 - \lambda \pi s)p_{b,C}^{co} \\
    V_s^U &= \pi b p_s^U + (1 - \pi b)p_{s,U}^{co} \\
    V_b^U &= -\pi s p_b^U - (1 - \pi s)p_{b,U}^{co}
\end{align*}
\]

Solving this system of equations for the \( V_s \), we obtain, for instance, that:

\[
p_C^s = -V_U^b = \frac{p_{b,U}^{co}(1 - \pi s) + p_{s,C}^{co}(1 - \lambda \pi b)\pi s}{1 - \lambda \pi b \pi s}.
\] (III.A.10)

This is exactly the same expression as in the original model (see (42) in the proof of Lemma 2), where the \( p_{j,k}^{co} \)s play the roles of \(-C^s, C^b, p^{co}\). Thus, we can proceed as in the baseline model to derive conditions on parameters under which the ACD, ICB, and ICS equilibrium regimes arise (as in Proposition 1).

It is worth stressing one difference with the baseline model when \( n_C \) is finite. In this case, connected buyers buy the asset from core dealers at a price strictly higher than \( p^{co} \) \( (p_{b,C}^{co} > p^{co}) \) while connected sellers sell the asset at a price strictly smaller \( (p_{s,U}^{co} < p^{co}) \). Thus, there are gains from trade between connected buyers and sellers (by trading among each other, they can trade at a price strictly between \( p_{s,C} \) and \( p_{b,C} \)). This possibility does not change the main mechanisms in our model. However, it makes the model more complex as it adds new possible equilibrium regimes for some values of the parameters (like when...
$\lambda > \frac{1}{2}$). The set of parameters for which these new regimes arise “shrinks” as $n^C$ increases and is empty as $n_C$ goes to infinity (as in our baseline model).

## B A model of matching in the periphery

In this section we sketch a model in which connected and unconnected dealers in the periphery get matched sequentially. The steady state matching probabilities correspond exactly to the stationary measures of the different states in our initial model. We conclude from this extension that our model with sequential arrival of dealers can be reinterpreted as a matching model.

### B.1 Setup

Consider the following discrete-time process:

- In each period $t$, a new cohort of $\alpha^{pe}$ sellers and $1 - \alpha^{pe}$ buyers arrive on the market.

- The buyers (resp. sellers) who arrived at time $t - 1$ and were not matched in $t - 1$ are matched to the dealers who arrive at time $t$. Each buyer (seller) has a probability $\pi_s$ ($\pi_b$) of being matched with a seller (buyer), where $\pi_b$ and $\pi_s$ are defined as in the main model.

- Define $\mu_i^t$ the number of dealers who enter the market at time $t$ and are:

  - $i = 1$: Connected sellers who make an offer or go to the core.
  - $i = 2$: Connected sellers who accept an offer.
  - $i = 3$: Unconnected sellers who make an offer.
  - $i = 4$: Unconnected sellers who accept an offer.
  - $i = 5$: Connected buyers who make an offer or go to the core.
  - $i = 6$: Connected buyers who accept an offer.
  - $i = 7$: Unconnected buyers who make an offer.
  - $i = 8$: Unconnected buyers who accept an offer.
B.2 Solution

We want to solve for the steady-state values of the $\mu_t^i$. Note that these values depend on the type of equilibrium we assume. We focus on the case $\alpha^{pe} > 1/2$ and the ACD Equilibrium, but the reasoning easily extends to the other cases.

The mass of connected sellers who make an offer or go to the core is the total number of new connected sellers minus those who accept an offer from the previous cohort:

$$\mu_1^t+1 = (1 - \lambda)\alpha^{pe} - \mu_2^{t+1}.$$  \hspace{1cm} (III.B.1)

The mass of connected sellers who accept an offer is equal to the mass of buyers who make an offer acceptable by connected sellers, which is $\mu_7^t$, times the probability that they find a connected seller, which is $1 - \lambda$. This gives:

$$\mu_2^{t+1} = \mu_7^t (1 - \lambda).$$  \hspace{1cm} (III.B.2)

The mass of unconnected sellers who make an offer is the total number of unconnected sellers minus those who accept an offer from the previous cohort:

$$\mu_3^{t+1} = \lambda \alpha^{pe} - \mu_4^{t+1}.$$  \hspace{1cm} (III.B.3)

The mass of unconnected sellers who accept an offer is equal to the mass of buyers who make an offer, which is $\mu_5^t + \mu_7^t$, times the probability that they find an unconnected seller, which is $\lambda$. This gives:

$$\mu_4^{t+1} = (\mu_5^t + \mu_7^t)\lambda.$$  \hspace{1cm} (III.B.4)

The case of buyers is symmetric, except that we need to take into account that sellers find
buyers only with a probability \( \pi_b = \frac{1 - \alpha_{pe}}{\alpha_{pe}} \). We obtain:

\[
\begin{align*}
\mu_5^{t+1} &= (1 - \lambda)(1 - \alpha_{pe}) - \mu_6^{t+1} \quad \text{(III.B.5)} \\
\mu_6^{t+1} &= \mu_3^t \pi_b (1 - \lambda) \quad \text{(III.B.6)} \\
\mu_7^{t+1} &= \lambda (1 - \alpha_{pe}) - \mu_8^{t+1} \quad \text{(III.B.7)} \\
\mu_8^{t+1} &= \lambda \pi_b (\mu_1^t + \mu_3^t). \quad \text{(III.B.8)}
\end{align*}
\]

The limit when \( t \to +\infty \) of the masses \( \mu_i^t \) can be obtained by solving the system \( \{ \mu_i^{t+1} = \mu_i^t \}_{1 \leq i \leq 8} \). The solution we obtain is exactly the same as the stationary measures defined in the paper. The correspondence between the two models comes from two reasons. First, the \( \mu_i \)s are ultimately determined by the same market clearing constraints. Second, \( \pi_b \) and \( \pi_s \) in the main model are defined in such a way that the proportion of periods in which a seller arrives on the market is exactly \( \alpha_{pe} \). Thus, the model with sequential arrival of dealers can be interpreted as a model in which dealers of different types are matched with each other over time.

### C Convex inventory costs

This section develops a variant of the model in which dealers face convex inventory costs and core dealers face no inventory shock between \( t = 2 \) and \( t = 3 \). We show that this model is isomorphic to our initial framework. Hence, our specification of inventory shocks can be thought of as equivalent to assuming convex inventory costs.

Assume that (i) dealers bear convex holding cost, \( C(|z_3|) \), with \( C(0) = 0 \), \( C'(x) > 0 \) for \( x > 0 \), \( C'(0) = 0 \) and \( C''(x) > 0 \) for \( x > 0 \) (strictly convex cost). We show below that there is a specification of \( C(\cdot) \) in which this approach is formally equivalent to our initial model.

Consider peripheral dealers first. If the peripheral dealer starts with a long position \( (z_3 = +1) \) and does not trade, he bears a cost \( C(1) \). If instead the dealer sells one unit, he bears a cost of zero. If the peripheral dealer starts with a short position \( (z_3 = -1) \) and does not trade, he bears a cost \( C(1) \). If instead, the dealer sells one unit, he bears a cost of zero.
Thus, the analysis for peripheral dealers is identical (defining \( C^s = C^b = C \equiv C(1) \)).

Now consider core dealers. The expected payoff of a core dealer when she buys \( q_{i2} \) units in the core market is:

\[
E(\Pi_i) = (E(v) - p^{co})q_{i2} - C(|z_{i0} + q_{i2}|),
\]

where \( E(v) = 0 \) is the expected payoff of the asset (normalized to zero in the paper). Let \( \iota(x) = -1 \) if \( x > 0 \) and \( \iota(x) = +1 \) if \( x < 0 \). Then, the optimal demand of core dealer \( i \) solves:

\[
p^{co} = C'(|z_{i0} + q_{i2}^*|) \times \iota(z_{i0} + q_{i2}^*), \quad \text{if } p^{co} \neq C'(|z_{i0}|),
\]

and is \( q_{i2}^* = 0 \) if \( p^{co} = C'(|z_{i0}|) \). Now let \( z^* = z_{0}^{co} + \Delta \) where \( \Delta \) is defined as in the paper. As core dealers are price takers in the core market, their marginal inventory holding cost must be equalized after trading at date 2. Moreover, by market clearing, the sum of their post trading positions must be equal to \( z^* \). It follows that post trading: \( z_{i0} + q_{i2}^* = z^* \) and therefore the equilibrium price is

\[
p^{co*}(z^*) = C'(|z^*|) \times \iota(z^*) \quad \text{if } z^* \neq 0 \quad \text{and } p^{co*} = C'(0) = 0 \quad \text{if } z^* = 0. \quad (\text{III.C.1})
\]

Observe that \( p^{co*}(z^*) \) decreases with core dealers’ aggregate inventory as usual in inventory holding costs models (\( \frac{\partial p^{co*}(z^*)}{\partial z^*} = -C'(|z^*|) < 0 \)).

Now, define the marginal inventory holding cost function as

\[
C'(|z^*|) = (-C + 2C\Phi(-\kappa^{-1}z^*))\iota(z^*), \quad (\text{III.C.2})
\]

where \( \Phi(.) \) and \( \kappa \) are defined as in the paper. As \( \Phi(0) = \frac{1}{2} \), it immediately follows that \( C'(|z^*|) > 0 \) for \( z^* > 0 \) and \( C'(0) = 0 \) for \( z^* = 0 \). Moreover,

\[
C'(|z^*|) = \frac{\partial \Phi(-\kappa^{-1}z^*)}{\partial z^*} > 0.
\]

Thus, \( C(x) \) is convex for \( x > 0 \). Last, the price in the core market (equation (III.C.1)) can

\[^{14}\text{In this case, we have symmetric inventory holding costs for long and short positions.}\]
then be written as:

\[ p^{co^*} = \Phi(\kappa^{-1}z^*)C - (1 - \Phi(\kappa^{-1}z^*))C, \]

which exactly corresponds to our model (see Lemma 1 in the paper) for \( C^s = C^b = C \).

This shows that the results of our model hold if we assume that (i) dealers bear convex inventory holding costs specified as in (III.C.2). More generally, as the only important property for our results is \( \frac{\partial p^{co^*}(z^*)}{\partial z^*} < 0 \), our results hold qualitatively for any convex cost function for dealers.

## D Bilateral transactions using double auctions

### D.1 Summary

In this section we consider the robustness of our results to using other trading mechanisms than take-it-or-leave-it offers. Note that when a buyer and a seller meet, they don’t know each other’s types, a situation of two-sided asymmetric information. As a result, we cannot use the standard Nash bargaining solution common in the search literature. Instead, the natural generalization of take-it-or-leave-it offers in this context is a double auction: both the buyer and the seller name a price, and a trade occurs if the seller’s price is higher than the buyer’s price, at some weighted average of the two prices.\(^{15}\)

The following analysis proves two claims:

1) The equilibrium regimes that can be obtained with double auctions only lead to trades that correspond to the ACD, ICS, and ICB regimes. Hence, assuming take-it-or-leave-it offers does not lead us to neglect other possible equilibrium allocations.

2) When a seller and a buyer meet, it is not necessarily in their common interest to use a double auction rather than a take-it-or-leave-it offer. On the contrary, we show that in the ACD equilibrium a pair of traders would be made worse off by using double auctions rather than take-it-or-leave-it offers.

Point 2) is important, as it shows that our inefficiency results are not an artifact of the take-it-or-leave-it mechanism. Actually, as we will show below, when two traders meet, we

\(^{15}\)See Duffie, Malamud, and Manso (2014) for a recent paper using double auctions in a search environment.
have an exchange game with two-sided asymmetric information in which there may be no
gains from trade between buyers and sellers. It is known from Myerson and Satterthwaite
(1983) that generically ex-post efficiency cannot be guaranteed in such an environment. That
is, there is no mechanism that ensures that the asset will be allocated to the buyer if and
only if his valuation is larger than the seller’s. Our result that double auctions can lead
to worse outcomes than take-it-or-leave-it offers, and also lead to additional problems such
as multiplicity of equilibria, is well in line with the general analysis of Satterthwaite and
Williams (1989).

Finally, while we are able to prove that there is no obvious generalization of the take-it-
or-leave-it offers that always leads to better outcomes, it is not clear at all that the take-it-or-
leave-it offer is the best mechanism. To our knowledge, which type of mechanism is optimal
in such a dynamic two-sided asymmetric information environment is an open question in the
mechanism design literature, and is beyond the scope of this paper.

D.2 Setup

Take a given equilibrium in the periphery with given values of $V^k_j$, and consider a given
period $t$. With no loss of generality, assume that in period $t$ we have a seller and in $t + 1$ a
buyer arrives (if a seller arrives then the outcome is trivial). We consider the possibility that
in period $t$ instead of the seller making a take-it-or-leave-it offer we run a double-auction.
Importantly, we do not change the rest of the game, so that the buyer’s outside option is still
given by the equilibrium value of $V^k_b$.

The auction works as follows. The seller (resp. buyer) names a price $s$ (resp. $b$). If $b < s$
then no trade occurs. If $b \geq s$ the seller sells one unit to the buyer, in exchange for a price
$p = kb + (1 - k)s$, with $k \in [0, 1]$. As is well known, the case $k = 0$ is strategically equivalent
to a take-it-or-leave-it offer made by the seller, as in our original model.

The economic environment can be thought of as follows: there is a seller with a valuation
$v_s \in \{v^L_s, v^H_s\}$ for the asset, with $v^L_s = -C^S$ (unconnected sellers) and $v^H_s = p^o$ (connected
sellers). Similarly, the buyer has a valuation $v_b \in \{v^L_b, v^H_b\}$, with $v^L_b = -V^C_b$ (connected
buyers) and $v^H_b = -V^U_b$ (unconnected buyers). The ranking of the different valuations
depends on the equilibrium type. $v^L_s$ is always the lowest valuation, and we also have $v^L_b < v^H_b$.
and $v_b^L \leq v_s^H$ (no gains from trade between connected dealers). The other comparisons depend on whether $\alpha > 1/2$ and whether the equilibrium is ACD, ICB, or ICS.

### D.3 Possible equilibrium types

An equilibrium is a 4-uple $\{p_b^L, p_b^H, p_s^L, p_s^H\}$ such that a type $k \in \{L, H\}$ buyer (resp. seller) names a price $p_b^k$ (resp. $p_s^k$). In case in equilibrium a type of dealer prefers his outside option to any offer, we assume that he names a price equal to his valuation (this rules out “trivial” equilibria that are known to exist in such games, e.g., all sellers name $C^B$ and all buyers name $-C^S$, in which case there is an equilibrium with no trade).

It is easy to show that in any equilibrium we must have $p_b^H \geq p_b^L$ and $p_s^L \leq p_s^H$. Similarly to our solution method in the original game, we can consider different types of equilibria, as follows. Define an equilibrium type by $(T_{sL,bH}, T_{sL,bL}, T_{sH,bH}, T_{sH,bL})$, where $T_{sk,bk'} = 1$ if and only if $sk$ and $bk'$ trade together in equilibrium, that is, $p_b^k \geq p_b^{k'}$. This defines in principle $2^4 = 16$ types of equilibria. However, this number can be reduced to 6 by observing that:

(i) If $T_{sL,bL} = 1$ then $T_{sL,bH} = 1$; (ii) If $T_{sH,bH} = 1$ then $T_{sL,bH} = 1$; (iii) If $T_{sH,bL} = 1$ then $T_{sH,bH} = T_{sH,bL} = T_{sL,bL} = 1$.

We then obtain the following possible equilibrium configurations for the double auction:

- **Maximum Trade**: $(T_{sL,bH}, T_{sL,bL}, T_{sH,bH}, T_{sH,bL}) = (1, 1, 1, 1)$: Trade occurs with probability 1.
- **ACD (Double Auction)**: $(T_{sL,bH}, T_{sL,bL}, T_{sH,bH}, T_{sH,bL}) = (1, 1, 1, 0)$: Trade occurs unless the buyer has a low valuation and the seller a high valuation (as in our original ACD equilibrium).
- **ICB (Double Auction)**: $(T_{sL,bH}, T_{sL,bL}, T_{sH,bH}, T_{sH,bL}) = (1, 0, 1, 0)$: Trade occurs unless the buyer has a low valuation, as in our original ICB equilibrium.
- **ICS (Double Auction)**: $(T_{sL,bH}, T_{sL,bL}, T_{sH,bH}, T_{sH,bL}) = (1, 1, 0, 0)$: Trade occurs unless the seller has a high valuation, as in our original ICS equilibrium.
- **Inactive Connected Dealers**: $(T_{sL,bH}, T_{sL,bL}, T_{sH,bH}, T_{sH,bL}) = (1, 0, 0, 0)$: Trade occurs only between low valuation sellers and high valuation buyers.
- **No Trade**: $(T_{sL,bH}, T_{sL,bL}, T_{sH,bH}, T_{sH,bL}) = (0, 0, 0, 0)$: No trade occurs.
Claim 1: The only possible equilibrium configurations for the double auction are ACD, ICB, ICS.

Proof: Consider the Maximum Trade configuration. Both types of sellers make offers that are accepted by the low-type buyer. Hence they choose the highest price below $p^L_b$, which gives $p^L_s = p^H_s = p^L_b$. Symmetrically, we obtain $p^L_b = p^H_b = p^H_s$. Thus, all prices are equal to the same price $p^*$. For such an equilibrium to be possible, we need that no type of player deviates. The only possible deviation here is for a player to not make an offer. For such deviations to be unprofitable a necessary and sufficient condition is to have $p^* \geq v^H_s$ and $p^* \leq v^L_b$. Hence, we need $v^L_b \geq v^H_s$. It turns out that we have $v^L_b = v^H_s$ when the ACD Equilibrium or the ICB Equilibrium obtain in the periphery. However, this equality is not robust to introducing any small probability that a trader on the uncrowded side doesn’t find a counterparty (see Section F), in which case we always have $v^L_b < v^H_s$.

Consider the No Trade configuration. Nobody trades in this configuration, so that each type bids his reservation value for the asset: $p^k_i = v^k_i$. Since we always have $v^H_b > v^L_s$ this necessarily leads to some trades, which contradicts the no-trade equilibrium. Such an equilibrium is thus impossible. Note that this is essentially ruled out by the assumption that a type that doesn’t trade bids his reservation value (which weakly dominates quoting a more extreme price). Otherwise a NT equilibrium always exists, for instance if sellers choose a price above $C^B$ and buyers a price below $C^S$.

Finally, consider the Inactive Connected Dealers configuration. In this equilibrium the high-type sellers and the low-type buyers don’t trade at all. We have $p^L_s = p^H_b = p^*, p^H_s = v^H_s$, $p^L_b = v^L_b$. We consider the possible deviations of all types of traders.

1. Low-type sellers trade with a probability $\lambda$ and get $U^L_s = \lambda p^* + (1 - \lambda) v^L_s$. They could deviate by not trading or offering $v^L_b$. To prevent this we need:

$$p^* \geq v^L_s$$  \hspace{1cm} (III.D.1)

$$\lambda(1-k)p^* + (1-\lambda)v^L_s \geq [1 - \lambda + \lambda(1-k)]v^L_b.$$  \hspace{1cm} (III.D.2)
2. High-type sellers do not trade and obtain \( v_s^H \). They could deviate and offer \( p^* \) or \( v_b^L \). To prevent this we need:

\[
p^* \leq v_s^H \quad (\text{III.D.3})
\]
\[
\lambda[kp^* + (1 - k)v_b^L] + (1 - \lambda)v_b^L \leq v_s^H. \quad (\text{III.D.4})
\]

3. Low-type buyers don’t trade and get zero. They could make an offer at price \( p^* \) or \( v_s^H \).

To prevent this we need:

\[
p^* \geq v_b^L \quad (\text{III.D.5})
\]
\[
\lambda[kv_s^H + (1 - k)p^*] + (1 - \lambda)v_s^H \geq v_b^L. \quad (\text{III.D.6})
\]

4. High-type buyers make an offer accepted with probability \( \lambda \) and get \( U_b^H = \lambda[v_b^H - p^*] \). They can deviate by not making an offer or making an offer at price \( v_s^H \). To prevent this we need:

\[
p^* \leq v_b^H \quad (\text{III.D.7})
\]
\[
(\lambda k + (1 - \lambda))v_b^H \geq (1 - \lambda)v_b^H + \lambda kp^*. \quad (\text{III.D.8})
\]

Conclusion: An ICD Double Auction Equilibrium is possible for a given \( k \) if a \( p^* \) exists that satisfies all conditions (III.D.1) to (III.D.10). In any regime we have \( v_b^L \geq v_s^L \) and \( v_b^L \leq v_s^H \). These observations and (III.D.5) imply (III.D.1), (III.D.3) implies (III.D.4), (III.D.5) implies (III.D.6). We are left with conditions (III.D.2), (III.D.3), (III.D.5), (III.D.7), (III.D.10).

The proof then depends on which type of equilibrium obtains in the periphery.

If \( \alpha \text{pe} < 1/2 \) then both in the ACD and in the ICB regime we have \( v_b^L = v_s^H \). Thus, (III.D.3) and (III.D.5) imply that \( p^* = v_b^L \). Plugging this into (III.D.2) gives \( v_s^L \geq v_b^L \), which is false.

If \( \alpha \text{pe} > 1/2 \) then (III.D.2) and (III.D.7) imply:

\[
\lambda(1 - k)v_b^H + (1 - \lambda)v_s^L \geq [1 - \lambda + \lambda(1 - k)]v_b^L. \quad (\text{III.D.9})
\]
As \( v_b^H \geq v_b^L \), this inequality implies in particular:

\[
\lambda v_b^H + (1 - \lambda) v_b^L \geq v_b^L.
\]  

(III.D.10)

Using the definitions of \( v_b^H, v_s^L, v_b^L \), this condition can be reexpressed as:

\[
\lambda(-V_U^b) + (1 - \lambda)(-C_s) \geq -V_C^b.
\]  

(III.D.11)

In words, this condition means that the equilibrium regime in the periphery is such that an unconnected seller prefers making an offer accepted by unconnected buyers only than making an offer accepted by both types of buyers. This is never the case when \( \alpha_{pe} > 1/2 \), as both in the ACD Regime and in the ICS Regime the unconnected sellers make offers that are accepted by both types of buyers. Hence, (III.D.11) is never satisfied when \( \alpha_{pe} > 1/2 \) (this can also be checked analytically).

This concludes the proof that the ICD Configuration can never be obtained in equilibrium, and hence that only the ACD, ICS, and ICB configurations may obtain. ■

D.4 Solution when \( \alpha_{pe} > 1/2 \) and the ACD Equilibrium obtains in the periphery

Consider the case in which \( \alpha_{pe} > 1/2 \) and the ACD Equilibrium obtains in the periphery. When a seller and a buyer meet, we have just shown that the double auction can lead to either an ACD, an ICS, or an ICB configuration. Solving the game leads to the following:

**Claim 2:** The ICS Configuration is an equilibrium for any value of \( k \in [0,1] \). The ACD Configuration is an equilibrium if and only if \( k = 0 \). The ICB Configuration is never an equilibrium.

When \( k = 0 \) the double auction is strategically equivalent to a take-it-or-leave-it offer made by the seller, so that naturally the configuration obtained with the take-it-or-leave-it offer also obtains in equilibrium. However, as soon as \( k > 0 \) the double auction leads to a
different outcome.

**Proof:**

1) Start with the ICB Configuration. Both types of sellers choose a price only accepted by high-type buyers: $p^L_s = p^H_s = p^H_b = p^*$. High-type buyers choose $p^*$, low-type buyers don’t trade and thus choose $p^L_b = v^L_b$. Consider the low-type sellers. They trade with probability $\lambda$ and obtain $\lambda p^* + (1 - \lambda)v^L_s$. Instead, they could make an offer at price $v^L_b$ and trade for sure, but at a lower price. For this deviation not to be profitable, we need:

$$\lambda(1 - k)p^* + (1 - \lambda)v^L_s \geq [1 - \lambda + \lambda(1 - k)]v^L_b.$$  

(III.D.12)

Consider high-type buyers. They get $v^H_b - p^*$. Their only profitable deviation is to not make an offer and get zero. For this not to be profitable we need:

$$p^* \leq v^H_b.$$  

(III.D.13)

Conditions (III.D.12) and (III.D.13) imply that we must have:

$$\lambda(1 - k)v^H_b + (1 - \lambda)v^L_s \geq [1 - \lambda + \lambda(1 - k)]v^L_b.$$  

(III.D.14)

Since $v^H_b \geq v^L_b$, this condition implies (III.D.10), which we have already shown is not satisfied when $\alpha^{pe} > 1/2$. Hence, an ICB Configuration cannot obtain in equilibrium.

2) Consider now the ACD Configuration. High-type sellers make offers that are accepted only by high-type buyers. The optimal such offer is $p^H_s = p^H_b$. Low-type sellers make offers that are accepted by low-type buyers as well, hence $p^L_s = p^L_b$. Conversely, high-type buyers choose offers accepted even by the high-type sellers, $p^H_b = p^H_s$, and low-type buyers choose offers that are accepted only by low-type sellers, $p^L_b = p^L_s$. Hence in any equilibrium we have two prices $p_L$ and $p_H$ with $p_L < p_H$ and $p^L_s = p^L_b = p_L$ and $p^H_s = p^H_b = p_H$. These prices need to satisfy:

$$p_L \leq p_H.$$  

(III.D.15)
We need to check for all possible deviations.

- Low-type sellers offer $p_L$. With probability $\lambda$ they meet a high-type buyer who offers $p_H$ and the transaction price is $kp_H + (1-k)p_L$. With probability $(1-\lambda)$ they meet a low-type buyer who offers $p_L$ and the transaction price is $p_L$. Hence, the low-type seller’s equilibrium payoff is:

$$U_s^L = \lambda[kp_H + (1-k)p_L] + (1-\lambda)p_L.$$  

A low-type seller could either not make an offer and get $v_s^L$, or choose $p_H$ and get $\lambda p_H + (1-\lambda)v_s^L$. We obtain the following two conditions:

$$\lambda[kp_H + (1-k)p_L] + (1-\lambda)p_L \geq v_s^L$$  

(III.D.16)  

$$(1-\lambda + \lambda(1-k))p_L \geq (1-\lambda)v_s^L + \lambda(1-k)p_H.$$  

(III.D.17)

- High-type sellers offer $p_H$. With probability $\lambda$ they meet a high-type buyer and the transaction price is $p_H$, with probability $1-\lambda$ they meet a low type buyer and the transaction doesn’t take place. Hence their equilibrium payoff is:

$$U_s^H = \lambda p_H + (1-\lambda)v_s^H.$$  

The possible deviations are to not make an offer and keep $v_s^H$ for sure, or make an offer at price $p_L$ and trade for sure, but at a lower expected price. For these deviations not to be profitable we need:

$$p_H \geq v_s^H$$  

(III.D.18)  

$$(1-\lambda + \lambda(1-k))p_L \leq (1-\lambda)v_s^H + \lambda(1-k)p_H.$$  

(III.D.19)

- Low-type buyers offer $p_L$. With probability $\lambda$ they meet a low type seller and trade at $p_L$, with probability $1-\lambda$ they meet a high-type seller and don’t trade. We obtain:

$$U_b^L = \lambda[v_b^L - p_L] + (1-\lambda) \times 0.$$  

The possible deviations are to not make an offer and get zero, or make an offer at price $p_H$.
and trade for sure at an expected price of \( \lambda[kp_H + (1 - k)p_L] + (1 - \lambda)p_H \). For these deviations to be unprofitable we need:

\[
    p_L \leq v_b^L, \quad (1 - \lambda + \lambda k)p_H \geq (1 - \lambda)v_b^L + \lambda kp_L. \tag{III.D.20}
\]

- High-type buyers offer \( p_H \). This offer is always accepted and they obtain:

\[
    U_H^b = v_b^H - [1 - \lambda + \lambda k]p_H - \lambda(1 - k)p_L.
\]

The possible deviations are to not make an offer or make an offer at price \( p_L \) that is only accepted by low-type sellers. We obtain:

\[
    v_b^H \geq [1 - \lambda + \lambda k]p_H + \lambda(1 - k)p_L \tag{III.D.22} \]

\[
    (1 - \lambda)v_b^H + \lambda kp_L \geq [1 - \lambda + \lambda k]p_H. \tag{III.D.23}
\]

- Conclusion: An ACD Double Auction Equilibrium is possible for a given value of \( k \) if we can find \( p_L \) and \( p_H \) that satisfy conditions (III.D.15) to (III.D.23). Moreover, any such price pair supports an equilibrium. The 9 conditions to satisfy can be substantially reduced. Observe first that (III.D.15) and (III.D.17) imply (III.D.16). In any configuration we have \( v_b^L \leq v_s^H \), hence (III.D.20) implies (III.D.19), and (III.D.18) and (III.D.20) imply (III.D.15). (III.D.15) and (III.D.18) imply (III.D.21). Finally, (III.D.23) implies (III.D.22).

Hence, we only need to satisfy (III.D.17), (III.D.18), (III.D.20), and (III.D.23), that is, \( p_H \geq v_s^H \), \( p_L \leq v_b^L \), and two conditions that require \( p_L \) to be large and \( p_H \) small. If these conditions can be satisfied then at least this has to be the case for \( p_L = v_b^L \) and \( p_H = v_s^H \).

Hence, necessary and sufficient conditions for an ACD Double Auction Equilibrium to exist are:

\[
    (1 - \lambda + \lambda(1 - k))v_b^L \geq (1 - \lambda)v_s^L + \lambda(1 - k)v_s^H \tag{III.D.24}
\]

\[
    (1 - \lambda)v_b^H + \lambda kv_b^L \geq [1 - \lambda + \lambda k]v_b^H. \tag{III.D.25}
\]
In the ACD Regime we have $v_b^H = v_s^H > v_b^L$, hence condition (III.D.25) holds only for $k = 0$. In $k = 0$ condition (III.D.24) becomes $v_b^L \geq (1 - \lambda)v_s^L + \lambda v_s^H$, which is equivalent to the condition for being in an ACD Regime in the first place and hence is true. Hence, the ACD Double Auction Equilibrium is possible if and only if $k = 0$.

3) Finally, consider the ICS Configuration. Low sellers choose $p_s^L = p^* = p_b^L$ which is accepted by both types of buyers, whereas both types of buyers choose the minimum price accepted by the low-type seller, that is $p_s^L = p_b^L = p^*$. High-type sellers do not trade and choose $p_s^H = v_s^H$. We check for all possible deviations.

- Low-type sellers get $p^*$ in equilibrium, their only possible deviation would be to not trade. Hence we need:

$$p^* \geq v_s^L.$$ \hspace{1cm} (III.D.26)

- High-type sellers do not trade, their only possible deviation would be to offer $p^*$. Hence, we need:

$$p^* \leq v_s^H.$$ \hspace{1cm} (III.D.27)

- Low-type buyers get $\lambda[v_b^L - p^*]$. They could deviate by either not trading or offering $v_s^H$ and trade with probability 1. We need:

$$p^* \leq v_b^L$$ \hspace{1cm} (III.D.28)

$$(1 - \lambda)v_b^L + \lambda kp^* \leq [1 - \lambda + \lambda k]v_s^H.$$ \hspace{1cm} (III.D.29)

- The conditions for high-type buyers are similar:

$$p^* \leq v_b^H$$ \hspace{1cm} (III.D.30)

$$(1 - \lambda)v_b^H + \lambda kp^* \leq [1 - \lambda + \lambda k]v_s^H.$$ \hspace{1cm} (III.D.31)

- Conclusion: An ICS Double Auction Equilibrium is possible for a given $k$ if a $p^*$ exists that satisfies all conditions (III.D.26) to (III.D.31). Observe that (III.D.31) implies (III.D.29) and (III.D.28) implies (III.D.30). Hence, we only need to satisfy (III.D.26), (III.D.27), (III.D.28), (III.D.29), (III.D.30), (III.D.31).
Using the ACD values of the $v_i^k$ these conditions reduce to:

$$\frac{(1 - \lambda)p^{co} - \lambda(1 - \pi_b)CS}{1 - \pi_b \lambda} \leq p^* \leq p^{co}.$$  \hspace{1cm} (III.D.32)

As this is always a non-empty interval, the ICS Double Auction Equilibrium can be obtained, with a continuum of possible values for the equilibrium price $p^*$. ■

Finally, we can consider the efficiency of the different equilibria. In the ACD Regime, we have $v_s^L < v_b^L < v_s^H = v_b^H$. However, as already noted, the last inequality is not robust to introducing even an infinitesimal probability that a peripheral buyers does not find a peripheral seller, so that more generically we have $v_s^L < v_b^L < v_s^H < v_b^H$. With this ranking, when a buyer and a seller meet, the efficient allocation is to have high type sellers trading with low type buyers only, and low type sellers trading with both types of buyers. This is achieved in the ACD Configuration, but not in the ICS Configuration. Hence the following claim:

**Claim 3:** Assume $k > 0$ (the double auction is not strategically equivalent to a take-it-or-leave-it offer). If $\alpha^{pe} > 1/2$ and the ACD Regime obtains in the periphery, when a buyer and a seller are matched together the take-it-or-leave-it mechanism implements the efficient allocation for this pair, whereas the double auction mechanism implements an inefficient allocation.

**E**  **Equilibria for the general case including $\lambda > 1/2$**

**E.1**  **Summary**

In this section, we relax the simplifying assumption that $\lambda \leq 1/2$ and provide the complete map of equilibria in that case. We make four points.

1) A necessary and sufficient condition for obtaining only equilibria of types ACD, ICB, and ICS is $\lambda \leq \lambda^*(\alpha^{pe})$. This condition is satisfied for any $\alpha^{pe}$ if and only if $\lambda \leq 1/2$, so that the assumption $\lambda \leq 1/2$ guarantees that we can focus on these equilibria. Fig. III.E.1 below
shows the region of parameters such that only equilibria of types ACD, ICB, and ICS can obtain.

2) When $\lambda > \lambda^*(\alpha^{pe})$ there are four new possible equilibrium types, denoted ACD*, ACD**, ICB*, ICS*. Each “starred” equilibrium type is a variant of the corresponding original type. The only difference is that in the starred equilibrium the unconnected dealers on the uncrowded side make offers that are only accepted by the unconnected dealers on the other side. Fig. III.E.2 below gives a graphical representation of all the equilibrium types. The presence of these new equilibrium types is intuitive: when $\lambda$ is large, there are many unconnected dealers and the incentives to make offers that target them increase. As a result, even traders on the crowded side may find this optimal, whereas when $\lambda \leq \lambda^*(\alpha^{pe})$ only traders on the uncrowded side do.

3) When $\lambda > 1/2$, depending on the value of $\alpha^{pe}$ the equilibrium will be of type ACD, ICS, ICB, ACD*, ACD**, ICS*, ICB*. Fig. III.E.3 shows the equilibrium map for different values of $\lambda$.

4) The economic forces driving our results remain the same in the new equilibrium types. In particular, a trader’s market power still derives from being on the uncrowded side, being connected, and the interaction between the two. We illustrate this point by solving for the prices in closed form in the ACD* equilibrium.
Figure III.E.1: The blue area corresponds to the regions of parameters in the \((\lambda, \alpha^{pe})\) space where only equilibria of types ACD, ICB, and ICS can obtain.

E.2 Analysis

We focus on the case \(z^{pe} \geq 0\). The case \(z^{pe} \leq 0\) is symmetric. Let \(\theta = \frac{C_b - \mu^{co}}{C^b + C^s}\). Moreover, define the following cutoff points

\[
\lambda^* = \frac{1 - \sqrt{1 - \pi_b}}{\pi_b} \tag{III.E.1}
\]

\[
\theta_1 = \frac{\lambda(1 - \lambda\pi_b)}{1 - \pi_b\lambda^2} \tag{III.E.2}
\]

\[
\theta_2 = \frac{\lambda(1 - \pi_b)}{1 - \lambda\pi_b(2 - \lambda)} \tag{III.E.3}
\]

\[
\theta_3 = \frac{1 - \lambda}{1 - \lambda^2\pi_b} \tag{III.E.4}
\]

Then, we have the following:

**Proposition.** If \(\lambda \leq \lambda^*\) the equilibrium is:

- ACD for \(p^{co} \leq (1 - \theta_2)C^b - \theta_2C^s\).
Figure III.E.2: Equilibrium configurations. An arrow going from one type of trader to another indicates that the former makes offers to the latter in equilibrium. In the “*” equilibria some links are unidirectional, which is emphasized by using a red arrow.
Figure III.E.3: Equilibrium map for $\lambda \in \{0.3, 0.6, 0.8\}$. 
ICS for $p^{co} > (1 - \theta_2)C^b - \theta_2 C^s$.

If $\lambda > \lambda^*$ the equilibrium is:

- ACD* for $p^{co} \leq (1 - \theta_1)C^b - \theta_1 C^s$.
- ICS* for $(1 - \theta_1)C^b - \theta_1 C^s < p^{co} \leq (1 - \theta_3)C^b - \theta_3 C^s$.
- ICS for $p^{co} > (1 - \theta_3)C^b - \theta_3 C^s$.

In order to prove this result, we consider all possible equilibrium types, including ACD and ICS.

### E.2.1 ACD Equilibrium

Recall that in the ACD Regime the strategies and payoffs are given by

\[
V^C_s = \frac{(1 - \pi_b \lambda)p^{co} + \pi_b \lambda(1 - \pi_s)C^b}{1 - \pi_s \pi_b \lambda} \tag{III.E.5}
\]

\[
V^U_s = \frac{\pi_b(1 - \pi_s \lambda)p^{co} - (1 - \pi_b)C^s}{1 - \pi_s \pi_b \lambda} \tag{III.E.6}
\]

\[
V^C_b = -\frac{(1 - \pi_s \lambda)p^{co} - \pi_s \lambda(1 - \pi_b)C^s}{1 - \pi_s \pi_b \lambda} \tag{III.E.7}
\]

\[
V^U_b = -\frac{\pi_s(1 - \pi_b \lambda)p^{co} + (1 - \pi_s)C^b}{1 - \pi_s \pi_b \lambda} \tag{III.E.8}
\]

and $p^C_s = -V^U_b$, $p^C_b = V^U_s$, $p^U_s = -V^C_s$, and $p^U_b = V^C_s$.

We derive the conditions under which dealers have no incentive to deviate from their equilibrium behavior. First, observe that we have $p^C_b \leq p^{co}$ and $p^C_s \geq p^{co}$. Hence, it is optimal for connected dealers to make offers. Consider an unconnected seller. He is better off not deviating if and only if:

\[
V^U_s > -\pi_b \lambda V^U_b - (1 - \pi_b \lambda)C^s. \tag{III.E.9}
\]

One can simplify this to:

\[
(C^s + p^{co})(1 - \lambda(2 - \pi_b \lambda)) > 0, \tag{III.E.10}
\]
Finally, an unconnected buyer is better off not deviating if and only if:

$$V_b^U > -\pi_s \lambda V_s^U - (1 - \pi_s \lambda) C^b.$$  \hspace{1cm} (III.E.12)

This can be simplified to

$$\frac{C^b - p^c^o}{C^b + C^s} > \frac{\lambda(1 - \pi_b)}{1 - \pi_b \lambda(2 - \lambda)}. \hspace{1cm} (III.E.13)$$

### E.2.2 ACD* Equilibrium

Consider now the variant ACD*, in which dealers follow the same strategies as in the ACD equilibrium, with the exception that unconnected sellers target unconnected buyers only. We thus postulate the following strategies:

$$p_b^C = V_s^U, \quad p_b^U = V_s^C, \quad p_s^C = -V_b^U, \quad \text{and} \quad p_s^U = -V_b^U.$$ 

This gives rise to the following system of equations:

$$V_b^C = -\lambda \pi_s V_s^U - (1 - \lambda \pi_s) p_c^o.$$  \hspace{1cm} (III.E.14)

$$V_b^U = -\pi_s V_s^C - (1 - \pi_s) C^b.$$  \hspace{1cm} (III.E.15)

$$V_s^C = -\lambda \pi_b V_b^U + (1 - \lambda \pi_b) p_c^o.$$  \hspace{1cm} (III.E.16)

$$V_s^U = -\pi_b \lambda V_b^U - (1 - \pi_b \lambda) C^s.$$  \hspace{1cm} (III.E.17)

The solution is given by:

$$V_s^C = \frac{(1 - \pi_b \lambda)p_c^o + \pi_b \lambda(1 - \pi_s)C^b}{1 - \pi_s \pi_b \lambda}.$$  \hspace{1cm} (III.E.18)

$$V_s^U = \frac{\pi_b \pi_s \lambda(1 - \pi_b \lambda)p_c^o + (1 - \pi_s)\pi_b \lambda C^b - (1 - \pi_s \pi_b \lambda)(1 - \pi_b \lambda) C^s}{1 - \pi_s \pi_b \lambda}.$$  \hspace{1cm} (III.E.19)

$$V_b^C = -\frac{((1 - \pi_b \lambda)\pi_b \pi_s^2 \lambda^2 + (1 - \pi_s \lambda)(1 - \pi_s \pi_b \lambda))p_c^o + (1 - \pi_s)\pi_b \pi_s \lambda^2 C^b - \pi_s \lambda(1 - \pi_s \pi_b \lambda)(1 - \pi_b \lambda) C^s}{1 - \pi_s \pi_b \lambda}. \hspace{1cm} (III.E.20)$$

$$V_b^U = \frac{-\pi_s (1 - \pi_b \lambda)p_c^o + (1 - \pi_s) C^b}{1 - \pi_s \pi_b \lambda}.$$  \hspace{1cm} (III.E.21)
Let us rule out profitable deviations. Unconnected sellers have no incentive to deviate if and only if:

\[ V_s^U > -\pi_b V_s^C - (1 - \pi_b)C^s. \]  

(III.E.22)

Note that, because \( \pi_s = 1 \), we have

\[ V_s^U = \frac{\pi_b \lambda (1 - \pi_b \lambda)p^{co} - (1 - \pi_b \lambda)(1 - \pi_b \lambda)C^s}{1 - \pi_b \lambda} \]  

(III.E.23)

\[ V_b^C = -\frac{((1 - \pi_b \lambda)\pi_b \lambda^2 + (1 - \lambda)(1 - \pi_b \lambda))p^{co} - \lambda(1 - \pi_b \lambda)(1 - \pi_b \lambda)C^s}{1 - \pi_b \lambda} \]  

(III.E.24)

so that \( C^b \) drops out completely. Rearranging (III.E.22) yields

\[ 0 > (1 - \lambda(2 - \lambda \pi_b))(p^{co} + C^s). \]  

(III.E.25)

Since \( p^{co} \geq -C^s \), this inequality is equivalent to \( \lambda > \lambda^* \). Similarly, connected buyers have no incentive to deviate if and only if:

\[ V_b^C > -p^{co} \]  

(III.E.26)

This collapses to \( p^{co} > -C^s \), which is always true. Next, connected sellers have no incentive to deviate if and only if

\[ V_s^C > p^{co}, \]  

(III.E.27)

which holds with equality, i.e. \( V_s^C = p^{co} \). Finally, unconnected buyers won’t want to change their strategy if and only if

\[ V_b^U > -\pi_s \lambda V_s^U - (1 - \pi_s \lambda)C^b. \]  

(III.E.28)

This simplifies to \( p^{co} > -\lambda V_s^U - (1 - \lambda)C^b \) because \( p^{co} = V_b^U \), and hence

\[ \frac{C^b - p^{co}}{C^b + C^s} > \frac{\lambda(1 - \lambda \pi_b)}{1 - \pi_b \lambda^2} \]  

(III.E.29)
Summing up, the ACD* regime is an equilibrium if and only if $\lambda > \lambda^*$ and (III.E.29) is satisfied.

**E.2.3 ICS Equilibrium**

Recall that prices and payoffs in the ICS Equilibrium are given by

\[
V^U_s = \frac{(1 - \pi_s \lambda)\pi_b p^{co} - (1 - \pi_b)C^s}{1 - \lambda \pi_s \pi_b}, \quad \text{(III.E.30)}
\]

\[
V^U_b = -\left(\frac{\lambda \pi_s \pi_b (1 - \pi_s \lambda) p^{co} - \pi_s \lambda (1 - \pi_b) C^s}{1 - \lambda \pi_s \pi_b}\right) + (1 - \pi_s \lambda) C^b, \quad \text{(III.E.31)}
\]

\[
V^C_b = -\frac{(1 - \pi_s \lambda) p^{co} - \pi_s \lambda (1 - \pi_b) C^s}{1 - \lambda \pi_s \pi_b}, \quad \text{(III.E.32)}
\]

\[
V^C_s = p^{co}. \quad \text{(III.E.33)}
\]

and $p^C_b = V^U_s$, $p^U_s = -V^C_b$, and $p^U_b = V^U_s$. Let us derive conditions for the absence of profitable deviations. First, consider unconnected buyers. Deviating to targeting connected sellers is suboptimal if and only if:

\[
V^U_b > -\pi_s V^C_s - (1 - \pi_s) C^b. \quad \text{(III.E.34)}
\]

Because $\pi_s = 1$, this simplifies to $V^U_b > -V^C_s$ and further to

\[
\frac{C^b - p^{co}}{C^b + C^s} < \frac{\lambda (1 - \pi_b)}{1 - \lambda \pi_b (2 - \lambda)}, \quad \text{(III.E.35)}
\]

Connected buyers have no incentive to deviate if and only if targeting unconnected sellers is better than directly buying in the core, i.e.,

\[
V^C_b > -p^{co}, \quad \text{(III.E.36)}
\]

which is always the case.

Next, consider connected sellers, who directly trade in the core in equilibrium. The absence of profitable deviations requires

\[
p^{co} > -\pi_b \lambda V^U_s + (1 - \pi_b \lambda) p^{co}. \quad \text{(III.E.37)}
\]
or more simply $p^{co} > -V_b^U$. This reduces to

$$\frac{C^b - p^{co}}{C^b + C^s} < \frac{\lambda(1 - \pi_b)}{1 - \lambda \pi_b (2 - \lambda)} \quad (\text{III.E.38})$$

Finally, consider unconnected sellers. Their strategy is optimal if and only if:

$$V_s^U > -\pi_b \lambda V_b^U - (1 - \pi_b \lambda) C^s. \quad (\text{III.E.39})$$

This expression simplifies to

$$\frac{C^b - p^{co}}{C^b + C^s} < \frac{1 - \lambda}{1 - \lambda^2 \pi_b}. \quad (\text{III.E.40})$$

In sum, the ICS equilibrium arises for

$$\frac{C^b - p^{co}}{C^b + C^s} < \min\left[\frac{1 - \lambda}{1 - \lambda^2 \pi_b}, \frac{\lambda(1 - \pi_b)}{1 - \lambda \pi_b (2 - \lambda)}\right]. \quad (\text{III.E.41})$$

The first term in the min is lower than the second one if and only if $(1 - \lambda \pi_b)(1 - \lambda (2 - \lambda \pi_b)) < 0$, that is, if $\lambda > \lambda^*$.  

### E.2.4 ICS* Equilibrium

In the ICS* regime the connected sellers go to the core, unconnected buyers target unconnected sellers, unconnected sellers target unconnected buyers, and connected buyers target unconnected sellers. The resulting payoffs are:

$$V_s^C = p^{co} \quad (\text{III.E.42})$$

$$V_b^C = -\lambda \pi_s V_s^U - (1 - \lambda \pi_s)p^{co} \quad (\text{III.E.43})$$

$$V_s^U = -\pi_b \lambda V_b^U - (1 - \lambda \pi_b) C^s \quad (\text{III.E.44})$$

$$V_b^U = -\pi_s \lambda V_s^U - (1 - \pi_s \lambda) C^b; \quad (\text{III.E.45})$$
with prices being $p^C_b = V^U_s$, $p^U_s = -V^U_b$, and $p^U_b = V^U_s$. Recall that a price can only be accepted if it is in the interval $[-C^s, C^b]$. Solving this system of equations yields

\begin{align*}
V^U_s &= \frac{(1 - \pi_s \lambda) \pi_b \lambda C^b - (1 - \pi_b \lambda) C^s}{1 - \pi_s \pi_b \lambda^2} \quad \text{(III.E.46)} \\
V^U_b &= -\frac{(1 - \pi_s \lambda) C^b - \pi_s \lambda (1 - \pi_b \lambda) C^s}{1 - \pi_s \pi_b \lambda^2} \quad \text{(III.E.47)} \\
V^C_b &= -\frac{\pi_s \pi_b \lambda^2 (1 - \pi_s \lambda) C^b - \pi_s \lambda (1 - \pi_b \lambda) C^s + (1 - \pi_s \pi_b \lambda^2)(1 - \pi_s \lambda) p^{co}}{1 - \pi_s \pi_b \lambda^2} \quad \text{(III.E.48)} \\
V^C_s &= p^{co}. \quad \text{(III.E.49)}
\end{align*}

In order to show this is indeed an equilibrium, we need to derive conditions under which no profitable deviations exist. As the only dealer that change strategy are unconnected sellers, we start with them. The absence of profitable deviations requires

\begin{align*}
V^U_s > -\pi_b V^C_b - (1 - \pi_b) C^s \quad \text{or} \\
\frac{C^b - p^{co}}{C^b + C^s} > \frac{1 - \lambda}{1 - \pi_b \lambda^2} \quad \text{(III.E.50)}
\end{align*}

which can be simplified to

\begin{align*}
\frac{C^b - p^{co}}{C^b + C^s} > \frac{1 - \lambda}{1 - \pi_b \lambda^2} \quad \text{(III.E.51)}
\end{align*}

Similarly, the absence of profitable deviations for unconnected buyers requires

\begin{align*}
V^U_b > -\pi_s V^C_s - (1 - \pi_s) C^b \quad \text{or} \\
\frac{\lambda (1 - \pi_b \lambda)}{1 - \pi_b \lambda^2} > \frac{C^b - p^{co}}{C^b + C^s} \quad \text{(III.E.52)}
\end{align*}

This can be simplified to

\begin{align*}
\frac{\lambda (1 - \pi_b \lambda)}{1 - \pi_b \lambda^2} > \frac{C^b - p^{co}}{C^b + C^s} \quad \text{(III.E.53)}
\end{align*}

Also, connected buyers will not deviate if and only if

\begin{align*}
V^C_b > -p^{co} \quad \text{or} \\
\frac{1 - \pi_b \lambda}{1 - \pi_b \lambda^2} > \frac{C^b - p^{co}}{C^b + C^s} \quad \text{(III.E.54)}
\end{align*}

which implies

\begin{align*}
\frac{1 - \pi_b \lambda}{1 - \pi_b \lambda^2} > \frac{C^b - p^{co}}{C^b + C^s} \quad \text{(III.E.55)}
\end{align*}
Finally, connected sellers have no incentive to deviate if and only if

\[ V_s^C > -\pi_b \lambda V_b^U + (1 - \pi_b \lambda) p^{co}, \]  

(III.E.56)

or \( p^{co} > -V_b^U \). Rearranging yields

\[ \frac{\lambda(1 - \pi_b \lambda)}{1 - \pi_b \lambda^2} > \frac{C^b - p^{co}}{C^b + C^s} \]  

(III.E.57)

Because \( \lambda \leq 1 \), we have found an equilibrium if and only if

\[ \frac{\lambda(1 - \pi_b \lambda)}{1 - \pi_b \lambda^2} > \frac{1 - \lambda}{1 - \pi_b \lambda^2} \]  

(III.E.58)

or alternatively

\[ (1 - \frac{\lambda(1 - \pi_b \lambda)}{1 - \pi_b \lambda^2}) C^b - \frac{\lambda(1 - \pi_b \lambda)}{1 - \pi_b \lambda^2} C^s < p^{co} < (1 - \frac{1 - \lambda}{1 - \pi_b \lambda^2}) C^b - \frac{1 - \lambda}{1 - \pi_b \lambda^2} C^s \]  

(III.E.59)

Notice from equation (III.E.58) that this requires \( \lambda(1 - \pi_b \lambda) > 1 - \lambda \), which is equivalent to \( \lambda > \lambda^* \).

### E.2.5 Other equilibrium candidates

For completeness, we can consider two equilibrium regimes that are also logically possible:

1) ACD**: Dealers follow the same strategies as in the ACD equilibrium with the exception that unconnected buyers target unconnected sellers only.

2) All dealers make offers that are only accepted by unconnected dealers on the other side.

We can apply the same method as above: compute the equilibrium payoffs under the postulated strategies, then consider deviations. Doing so, we are able to prove that: Equilibrium 2) can never obtain, even when \( \lambda > 1/2 \); The ACD** Equilibrium never obtains when \( \alpha^{pe} > 1/2 \). However, it can be obtained when \( \alpha^{pe} < 1/2 \), as it is the symmetric case of ACD* (unconnected buyers make offers accepted only by unconnected sellers, instead of the opposite). The proofs are omitted for brevity but can be shared upon request.
E.3 Discussion - ACD* Equilibrium

In the ACD* Equilibrium, we obtain the following prices: $p_b^U = p_s^U = p_s^C = p^C$ and $p_b^C = p^C - (1 - \pi_b \lambda)(C^s + p^C)$. As a result, the market power measures are given by:

\[
M_b^U = M_s^C = 1 \quad \text{(III.E.60)}
\]
\[
M_s^U = \lambda \pi_b \quad \text{(III.E.61)}
\]
\[
M_b^C = 1 + \lambda (1 - \lambda \pi_b) \frac{C^s + p^C}{C^b - p^C} \quad \text{(III.E.62)}
\]

In particular, like in Proposition 3, we obtain $M_b^U > M_s^U$, $M_b^C > M_s^C$, $M_s^C > M_s^U$, $M_b^C > M_b^U$. Both connectedness and being on the uncrowded side are sources of market power, even strictly so. The comparative statics with respect to $z_{pe}^0$ are also the same as in Proposition 3.

F Search frictions

F.1 Summary

The initial model does not feature any search friction in the sense that a dealer on the uncrowded side can meet a new dealer on the opposite side with probability 1 and at no cost. As a result, when either $\lambda = 0$ or $\alpha^{pe} = 1/2$ the equilibrium is Walrasian and there is no price dispersion. To study how the model is affected when a search friction is introduced, we study an extension in which the meeting probabilities are:

\[
\pi_b = \zeta \min \left( \frac{1 - \alpha^{pe}}{\alpha^{pe}}, 1 \right); \quad \pi_s = \zeta \min \left( \frac{\alpha^{pe}}{1 - \alpha^{pe}}, 1 \right). \quad \text{(III.F.1)}
\]

Our initial model studied the case $\zeta = 1$. In this section we relax this simplifying assumption and provide the complete map of equilibria for any $\zeta < 1$. In that case finding a counterparty leads to market power over that counterparty. Even in the case $\alpha^{pe} = 1/2$, an unconnected buyer who receives an offer knows that with probability $1 - \zeta$ he will not be able to find another seller, and is thus ready to accept a worse price than the one he could offer to another seller.
We make three points.

1) There are two new possible equilibrium types, ACD* and ACD**, in which all types of dealers behave as in the ACD Regime, except that unconnected dealers on the uncrowded side make offers that are only accepted by unconnected dealers on the other side. The set of parameters for which the ACD* (resp. ACD**) regime obtains is “between” those where the ACD and ICB (resp. ICS) regimes obtain. This is illustrated in Fig. III.F.1 below. Both sets become vanishingly small as $\zeta \to 1$ (i.e., there is no discontinuity in $\zeta = 1$).

2) For small values of $\zeta$ the size of $|z^{pe}|$ has less influence over which type of equilibrium obtains. Intuitively, when $\zeta < 1$ there is a new source of market power in the model, which is that the dealer making receiving an offer may not be able to find another counterparty if he rejects, even if he is on the uncrowded side. This source of market power becomes more important when $\zeta$ is lower. Conversely, being on the uncrowded side becomes less valuable when $\zeta$ is low, since the probability of finding a counterparty is small. In the limit, as $\zeta \to 0$ the equilibrium type does not depend on $\alpha^{pe}$ anymore.

3) While $\zeta < 1$ introduces a new source of market power, the other two sources of market power still play a role and the main properties of the model are conserved.

**F.2 Analysis**

Consider any value $\zeta < 1$ and define the following quantities:

\[
\omega_b = \frac{\lambda(1 - \pi_b)}{1 - \lambda \pi_b (2 - \lambda \pi_s)}
\]  

\[
\omega_s = \frac{\lambda(1 - \pi_s)}{1 - \lambda \pi_s (2 - \lambda \pi_b)}
\]  

\[
\omega'_b = \frac{\lambda \pi_s (1 - \pi_b)}{1 - \lambda \pi_b \pi_s (2 - \lambda \pi_s)}
\]  

\[
\omega'_s = \frac{\lambda \pi_s (1 - \pi_s)}{1 - \lambda \pi_b \pi_s (2 - \lambda \pi_b)}.
\]

Observe that when $\zeta = 1$ and $\alpha^{pe} > 1/2$ we have $\omega_s = \omega'_s = 0$, whereas $\omega_b$ and $\omega'_b$ are both equal to the value of $\omega_b$ in the original model.

We then define the following thresholds:
Figure III.F.1: Equilibrium map for $\zeta \in \{0.1, 0.6, 0.8, 1\}$. 
The equilibrium surplus is then the following (see Section E for the definition of the equilibrium types ACD* and ACD**):

**Proposition.** The equilibrium regimes are:

- **ICB** for \( p^o < \bar{p}'_s \).
- **ACD** for \( p^o \in [\bar{p}'_s, \bar{p}_s] \).
- **ACD** for \( p^o \in [\bar{p}_s, \bar{p}_b] \).
- **ACD** for \( p^o \in (\bar{p}_b, \bar{p}'_b] \).
- **ICS** for \( p^o > \bar{p}'_b \).

In words, the difference with the case \( \zeta = 1 \) is that the new equilibrium type ACD* is “inserted” between ICB and ACD, whereas ACD** is inserted between ACD and ICS.

**F.3 Derivations**

The proofs in the main paper contain the equilibrium surplus of all types as a function of \( \pi_b \) and \( \pi_s \), so that these formula remain true even when \( \zeta < 1 \) and can be used here. We focus on the case \( \alpha^{pe} > 1/2 \), the case \( \alpha^{pe} < 1/2 \) obtains by symmetry (with ACD* being symmetric to ACD**).

**ACD Equilibrium:** The only change is in the expression of the conditions (43) and (45). The deviation for unconnected sellers gives \( p^o \geq \bar{p}_s \) and the deviation for unconnected buyers gives \( p^o \leq \bar{p}_b \).
ICS Equilibrium: Here again the expressions of surplus are the same as in the original model. Considering deviations:

Connected buyers must be better off trading in the periphery than in the core. This gives \( p^{co} \geq -C^s \), which is always true.

Connected sellers must be better off going to the core than making an offer in the periphery, which gives \( p^{co} > \bar{p}^b \).

Unconnected buyers must prefer making an offer targeting unconnected sellers to an offer targeting connected sellers. This gives \( p^{co} \geq (1 - \omega)C^b - \omega C^s \), with:

\[
\omega = \frac{\lambda(1 - \pi_b)}{1 - \lambda \pi_b (1 - \lambda) - \lambda \pi_b}. \tag{III.F.10}
\]

We can compute that \( \omega > \omega'_b \), so that \( p^{co} > \bar{p}^b \) implies \( p^{co} \geq (1 - \omega)C^b - \omega C^s \).

Unconnected sellers must prefer offers targeting connected buyers to offers targeting unconnected buyers. This gives \( p^{co} \geq (1 - \omega)C^b - \omega C^s \), with:

\[
\omega = \frac{1 - \lambda}{1 - \lambda^2 \pi_s \pi_b}. \tag{III.F.11}
\]

We have \( \omega > \omega'_b \) so that there is no incentive to deviate.

To conclude, we have shown that the ICS Equilibrium obtains if and only if \( p^{co} > \bar{p}^b \).

ACD** Equilibrium: This is symmetric to the ACD* equilibrium studied in Section E. The strategies are the following: \( p^C_b = V^U_s \), \( p^U_b = V^U_s \), \( p^C_s = -V^U_b \), and \( p^U_s = -V^C_b \). This gives rise to the following system of equations:

\[
V^C_b = -\lambda \pi_s V^U_s - (1 - \lambda \pi_s) p^{co} \tag{III.F.12}
\]
\[
V^U_b = -\lambda \pi_s V^U_s - (1 - \pi_s \lambda) C^b \tag{III.F.13}
\]
\[
V^C_s = -\lambda \pi_b V^U_b + (1 - \lambda \pi_b) p^{co} \tag{III.F.14}
\]
\[
V^U_s = -\pi_b V^C_b - (1 - \pi_b) C^s. \tag{III.F.15}
\]
The solution is given by

\[ V_b^C = \frac{-(1 - \lambda \pi_s)p^co - \lambda \pi_s(1 - \pi_b)C^s}{1 - \pi_s \pi_b \lambda} \]  

(III.F.16)

\[ V_b^U = \frac{-\pi_s \pi_b \lambda(1 - \lambda \pi_s)p^co + (1 - \pi_s \lambda)(1 - \pi_s \pi_b \lambda)C^b - \pi_s \lambda(1 - \pi_b)C^s}{1 - \pi_s \pi_b \lambda} \]  

(III.F.17)

\[ V_s^C = \frac{((1 - \lambda \pi_s)\pi_s \pi_b^2 \lambda^2 + (1 - \lambda \pi_b)(1 - \pi_s \pi_b \lambda))p^co + \lambda \pi_b(1 - \pi_s \lambda)(1 - \pi_s \pi_b \lambda)C^b - \pi_b \pi_s \lambda^2(1 - \pi_b)C^s}{1 - \pi_s \pi_b \lambda} \]  

(III.F.18)

\[ V_s^U = \frac{\pi_b(1 - \lambda \pi_s)p^co - (1 - \pi_b)C^s}{1 - \pi_s \pi_b \lambda}. \]  

(III.F.19)

We then consider the deviations.

Connected buyers could deviate and go directly to the core. This is unprofitable if and only if \( p^co \geq -C^s \), which is always true.

Unconnected buyers could deviate and make offers targeting connected sellers. We obtain that this is unprofitable if and only if \( p^co \geq \bar{p}_b \).

Connected sellers could deviate and go directly to the core. This is unprofitable if and only if \( p^co \leq \bar{p}_b \).

Unconnected sellers could deviate and make offers targeting unconnected buyers. We obtain that this is unprofitable if and only if \( p^co \leq \bar{p}_b' \).

To conclude, we have shown that the ACD** Equilibrium obtains if and only if \( p^co \in (\bar{p}_b, \bar{p}_b') \).

ICS* Equilibrium: The deviation for unconnected sellers gives \( p^co \leq (1 - \omega)C^b - \omega C^s \), with:

\[ \omega = \frac{1 - \lambda}{1 - \lambda^2 \pi_b \pi_s}. \]  

(III.F.20)

We have \( \omega > \omega_b \) so that \( p^co > \bar{p}_b \) implies \( p^co \geq (1 - \omega)C^b - \omega C^s \).

To conclude, we have shown that the ACD** Equilibrium obtains if and only if \( p^co \in (\bar{p}_b, \bar{p}_b') \).

ICS* Equilibrium: The deviation for unconnected sellers gives \( p^co \leq (1 - \omega)C^b - \omega C^s \), with:

\[ \omega' = \frac{\lambda \pi_s(1 - \lambda \pi_b)}{1 - \lambda^2 \pi_b \pi_s}. \]  

(III.F.21)

The deviation for connected sellers gives \( p^co \geq (1 - \omega')C^b - \omega' C^s \), with:

\[ \omega' = \frac{\lambda \pi_s(1 - \lambda \pi_b)}{1 - \lambda^2 \pi_b \pi_s}. \]  

(III.F.22)
Since $\lambda < 1/2$ we necessarily have $\omega' < \omega$, which implies that the conditions mentioned above are incompatible with each other. Hence, an ICS* Equilibrium can never obtain.

**Other equilibria:** Equilibria of type ICB cannot obtain when $\alpha^pe > 1/2$, the proof is omitted for brevity. This equilibrium obtains when $\alpha^pe < 1/2$, the proof being entirely symmetric with the treatment of ICS when $\alpha^pe > 1/2$. Equilibrium of type ACD* is symmetric to ACD**.

**Conclusion:** To conclude the proof, it is easily checked that the ordering of the different thresholds is correct: $\omega_b > \omega'_b$, so that $\bar{p}_b < \bar{p}'_b$; by symmetry we obtain $\bar{p}_s > \bar{p}'_s$; and finally $\bar{p}_s < \bar{p}_b$.

### F.4 Discussion - ACD** Equilibrium

When $\alpha^pe > 1/2$ and the ACD** Equilibrium obtains, we get the following market power measures (remember that $\pi_s = \zeta$ and $\pi_b \leq \zeta$):

\[
M^U_b = \lambda \zeta + \frac{\lambda \zeta (1 - \pi_b) p^co + C^s}{1 - \pi_b \lambda \zeta} C^b - p^co \tag{III.F.23}
\]

\[
M^U_s = \frac{\pi_b (1 - \lambda \zeta)}{1 - \pi_b \lambda \zeta} \tag{III.F.24}
\]

\[
M^C_b = 1 + \frac{\lambda \zeta (1 - \pi_b) p^co + C^s}{1 - \pi_b \lambda \zeta} C^b - p^co \tag{III.F.25}
\]

\[
M^C_s = \frac{1 - \lambda \zeta \pi_b (1 + \lambda (1 - \pi_b))}{1 - \lambda \zeta \pi_b} + \lambda \pi_b (1 - \lambda \zeta) \frac{C^b - p^co}{p^co + C^s} \tag{III.F.26}
\]

We can still show that $M^U_b > M^U_s$, $M^C_b > M^C_s$, $M^C_s > M^U_s$, $M^C_b > M^U_b$, as in Proposition 3. Hence, both being on the uncrowded side and being connected are still sources of market power. The comparative statics of these market power measures with respect to $\alpha^pe$ are also the same as in Proposition 3.

### References


